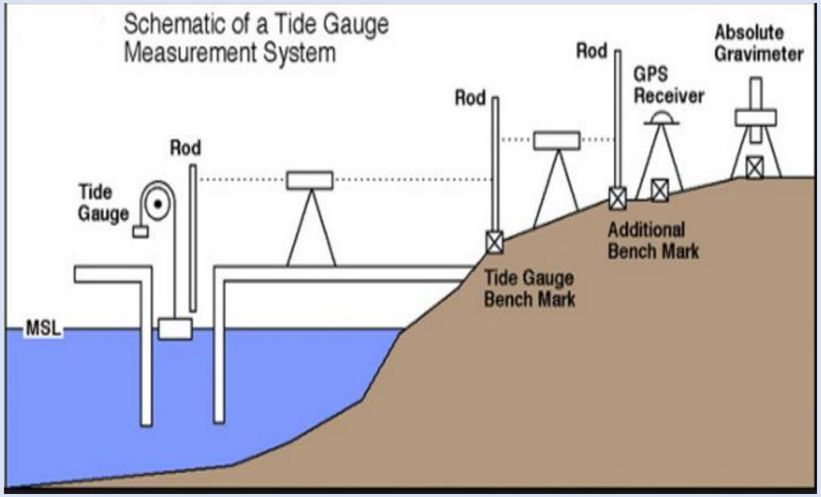
On Virtual Tide Gauge (VTG)

Zhigang Xu

Maurice Lamontagne Institute

A VTG Is Not An RTG

RTG (Real Tide Gauge)



50K to build a new RTG and 5k annually maintenance

VTG (Virtual Tide Gauge)

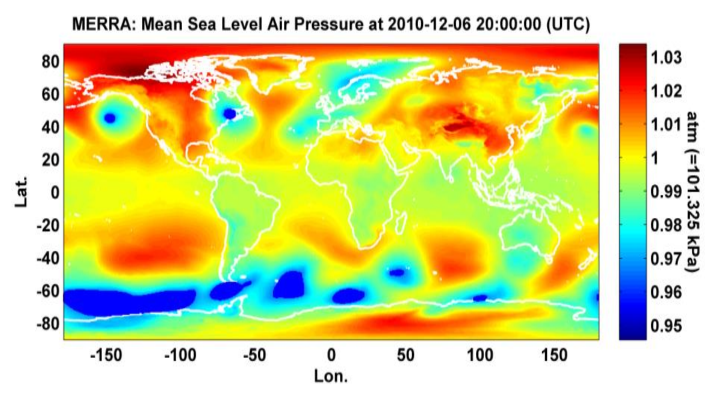
- A VTG does not resides in oceans, it is in a computer on internet.
- It is a good supplement or backup to RTG.

What is a Virtual Tidal Gauge (VTG)?

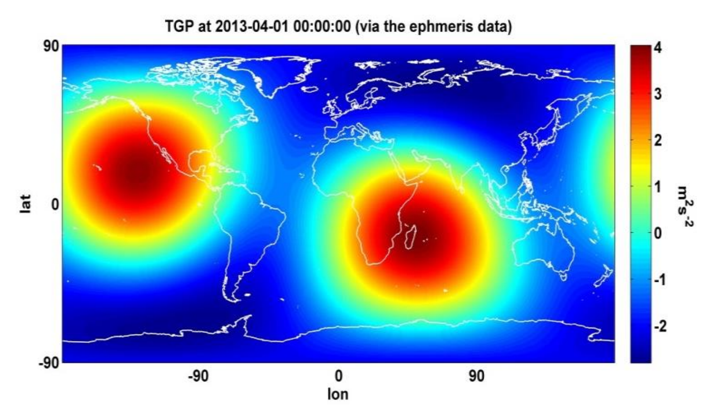
- A VTG is a mathematical transfer function to transfer astronomical and atmospheric global forcing fields to a time series of water level responses at a point of interest (POI);
- Its parameters are trained by observed data.

Two Forcing Fields

Atmospheric Forcing Field F_{atm}



Astronomic Forcing Field F_{tide}



$$\eta = \mathbf{G} * (\mathbf{F}_{atm} + \mathbf{F}_{tide})$$

$$\eta = \mathbf{C}\mathbf{s}$$

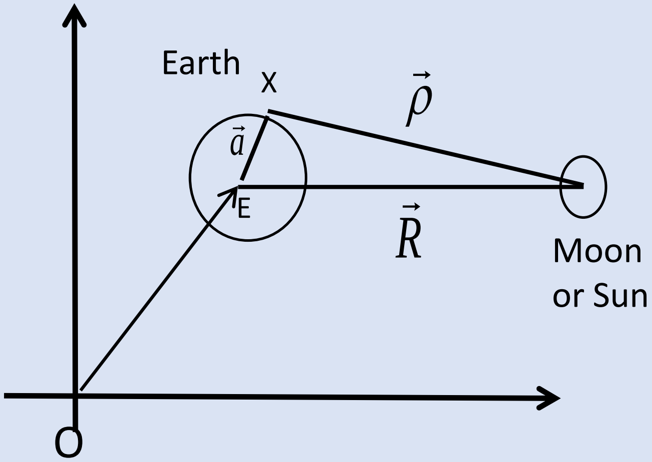
G All Source Green's Function (ASGF, Xu 2007); An MISO system.

↓

C Regression Model (Xu, 2015a,b) Regression Parameters

s

Tidal Forcing What's its definition?



$$\mathbf{B}_\mathbf{i}(\mathrm{E}) = \mu \frac{M_i \cdot 1}{R_i^2}\left(\frac{\vec{R}_i}{R_i}\right), \quad \mathbf{B}_\mathbf{i}(\mathrm{X}) = \mu \frac{M_i \cdot 1}{\rho_i^2}\left(\frac{\vec{\rho}_i}{\rho_i}\right)$$

$$\vec{\rho} = \vec{R} - \vec{a}$$

$$\frac{d^2}{dt^2}\overrightarrow{OE} = \sum_{i=1}^{n} \mathbf{B}_\mathbf{i}(E), \qquad \frac{d^2}{dt^2}\overrightarrow{OX} = \sum_{i=1}^{n} \mathbf{B}_\mathbf{i}(\mathrm{X}) + \mathbf{G} + \mathbf{F}$$

$$\frac{d^2}{dt^2}(\overrightarrow{OX} - \overrightarrow{OE}) = \sum_{i=1}^{n}\left(\mathbf{B}_\mathbf{i}(\mathrm{X}) - \mathbf{B}_\mathbf{i}(\mathrm{E})\right) + \mathbf{g} + \mathbf{F}$$

$$\frac{d^2}{dt^2}\overrightarrow{EX} = \sum_{i=1}^{n} \mathbf{T}_\mathbf{i}(E) + \mathbf{g} + \mathbf{F}$$

Tidal Forcing: the difference of gravitational forces on a point of interest and on the center of the earth exerted by astronomical bodies.

Time Rage of JPL ephemerides

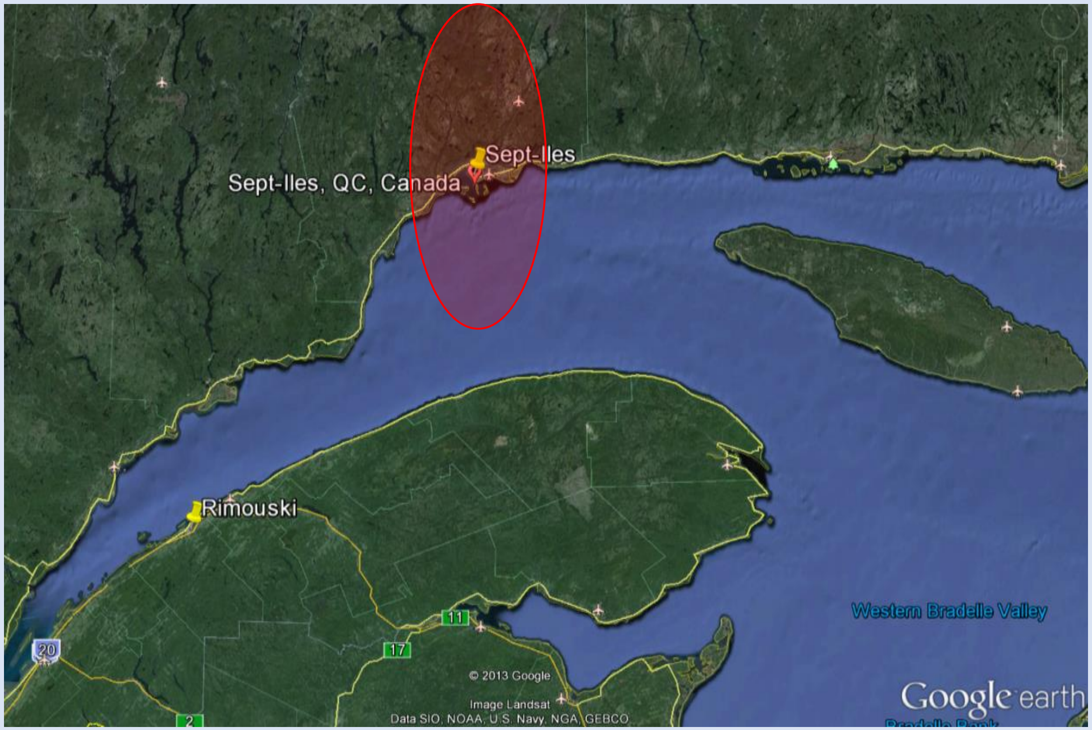
- De430.bin=
  [1549-12-31 00:00:00 to 2650-01-25 00:00:00]

- De431.bin=
  [-13001-08-31 00:00:00 to 17000-01-11 00:00:00]

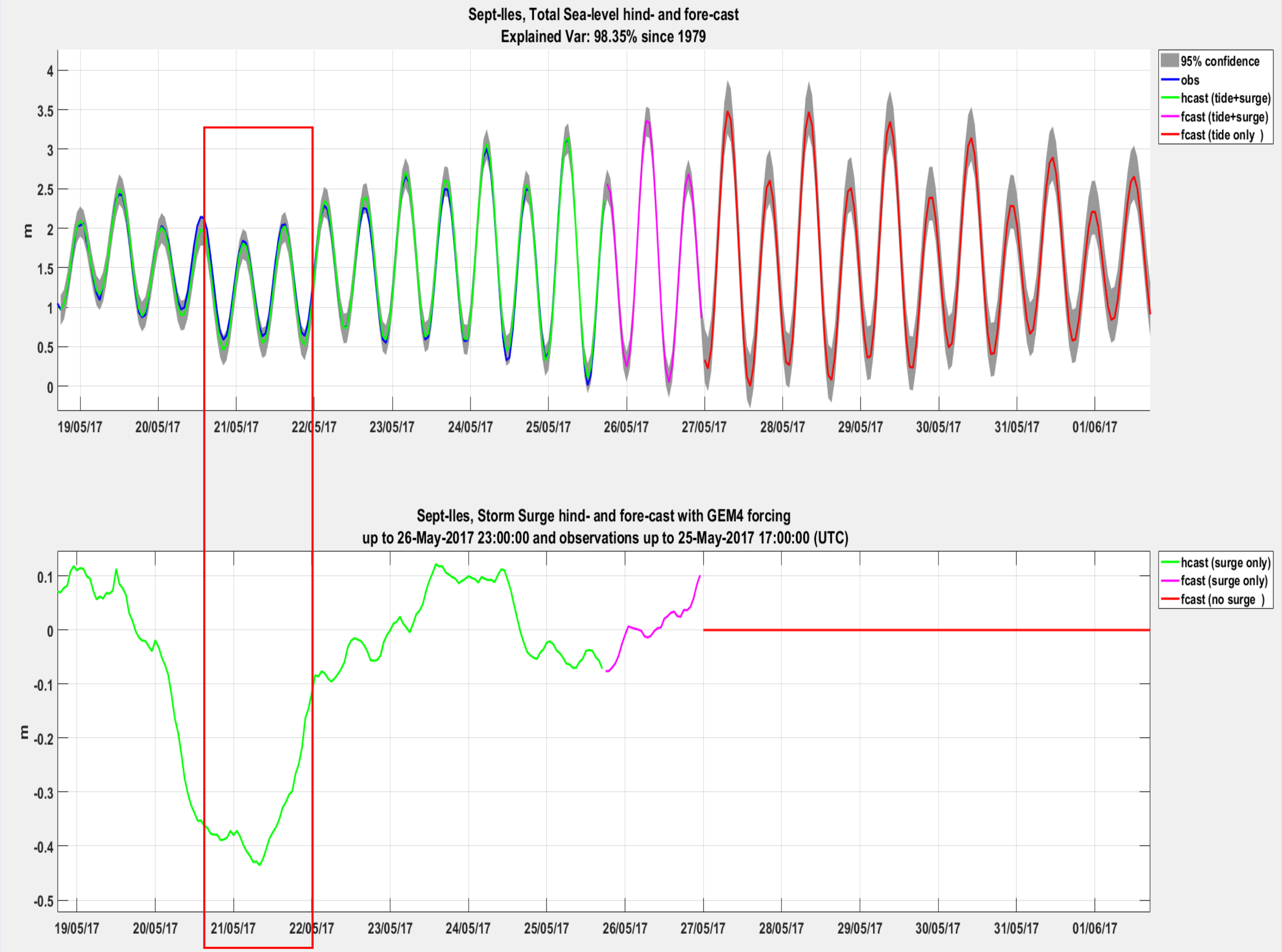
MERRA and GEM4 Model Solutions as Atmospheric Forcing Field

1. MERRA/MEERA2:
    - MODERN-ERA RETROSPECTIVE ANALYSIS FOR RESEARCH AND APPLICATIONS
    - NASA reanalysis using a major new version of the Goddard Earth Observing System Data Assimilation System Version 5 (GEOS-5).
    - Time Range: January 1, 1979 up to last month
2. GEM4 Real-time Forecast Data
    - Download Twice a day. 48 Hours forecast.
    - We archived it from January 1, 2016

A Demo VTG for Sept-Iles



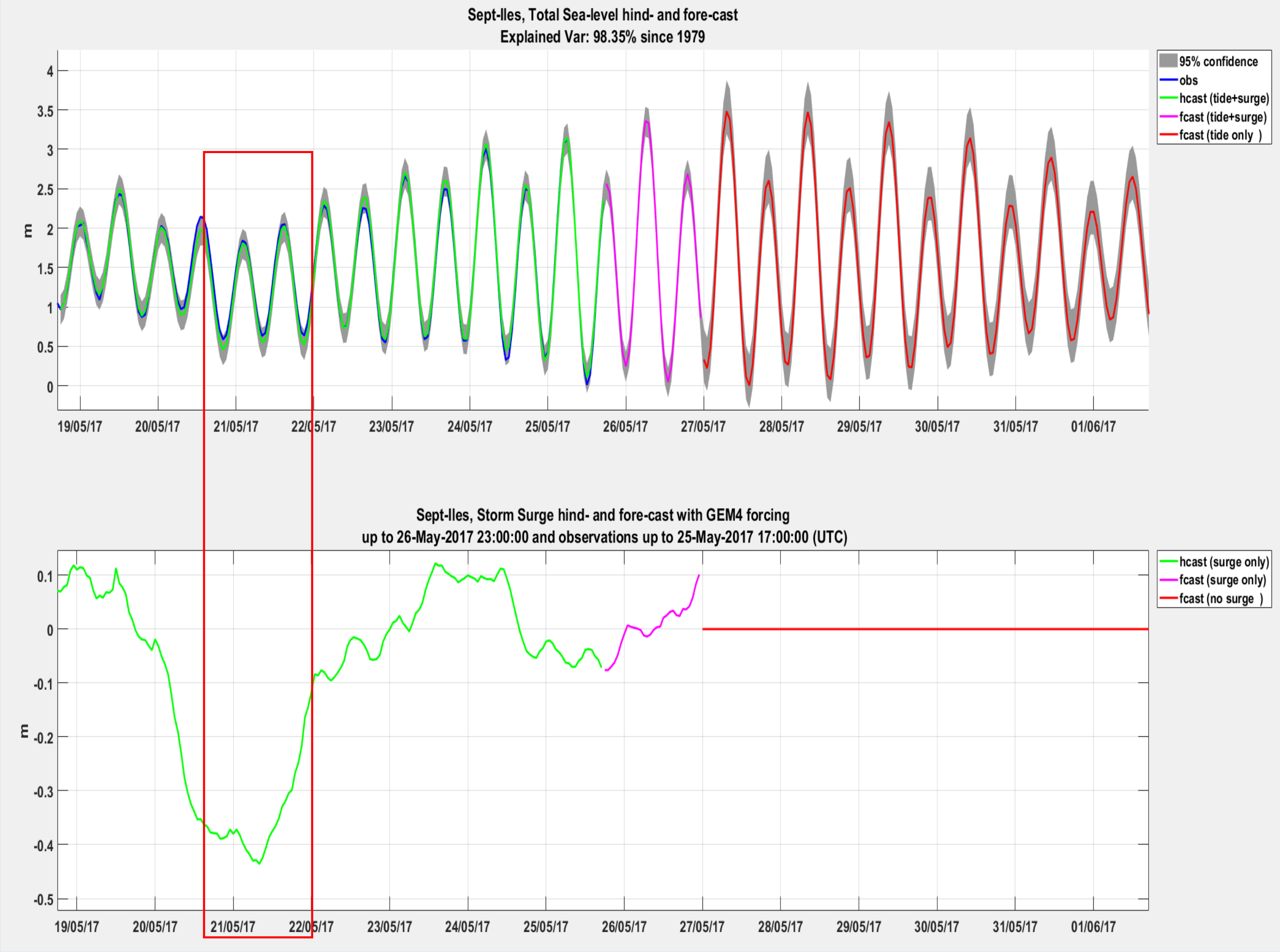
Demo VTG for Sept-Iles running in real-time



Sept-Iles, Total Sea-level hind- and fore-cast
Explained Var: 98.35% since 1979

Sept-Iles, Storm Surge hind- and fore-cast with GEM4 forcing
up to 26-May-2017 23:00:00 and observations up to 25-May-2017 17:00:00 (UTC)

Explained Variance: 98.35%



Sept-Iles, Total Sea-level hind- and fore-cast
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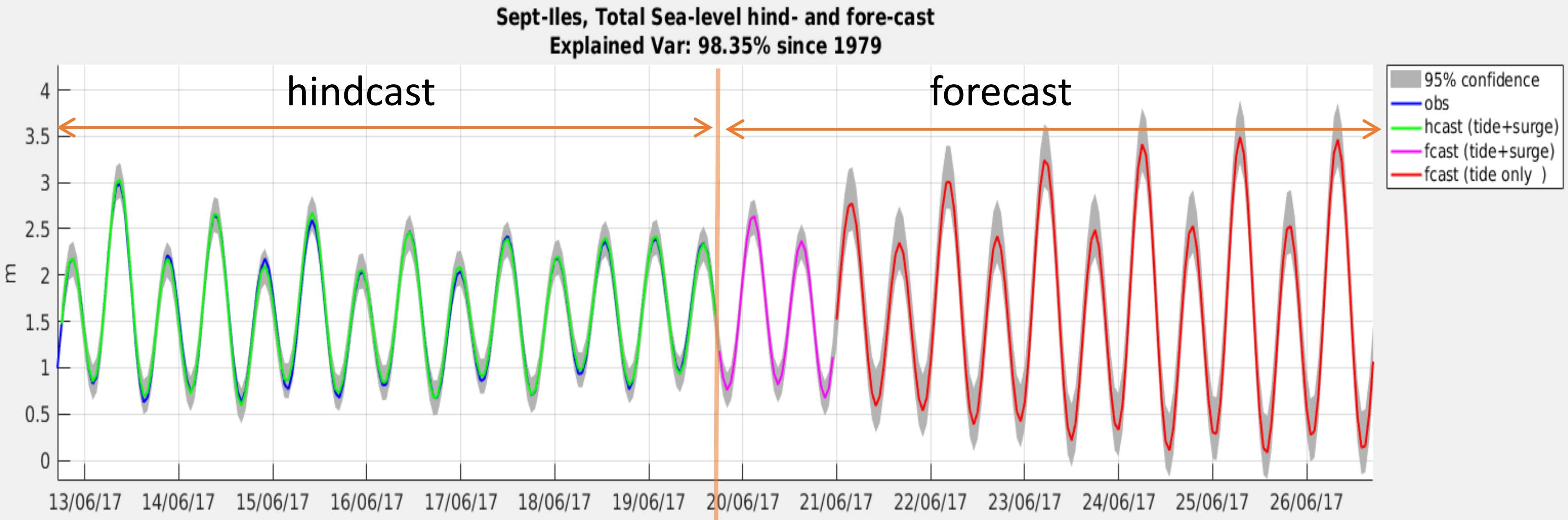
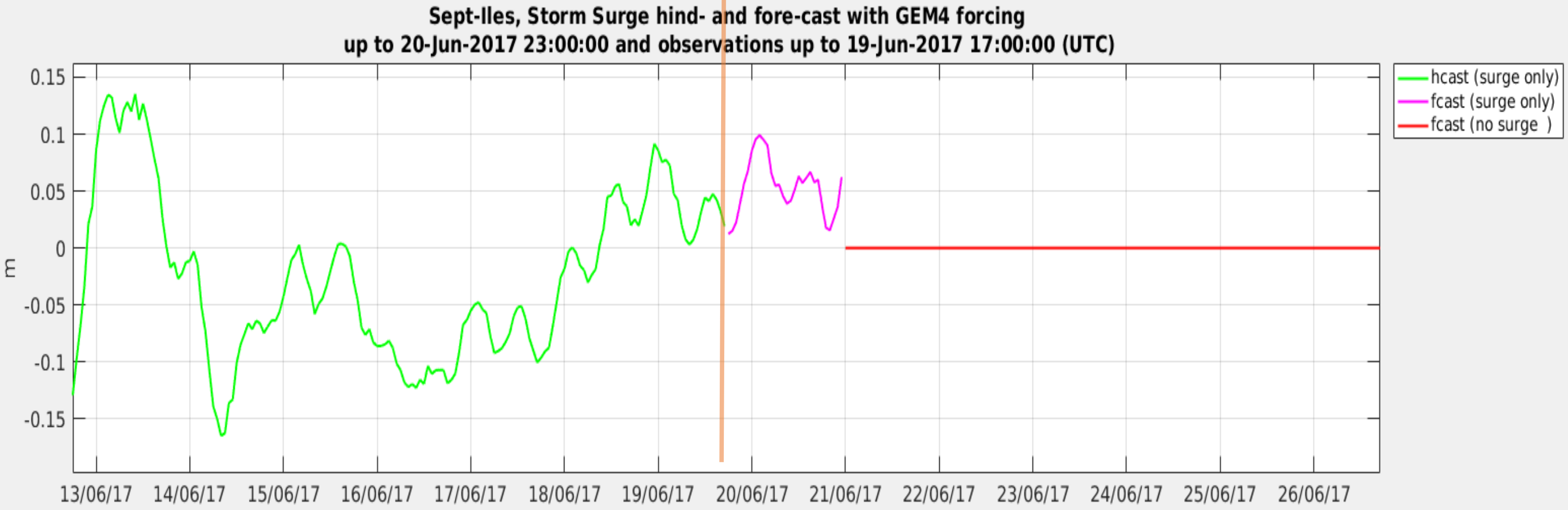


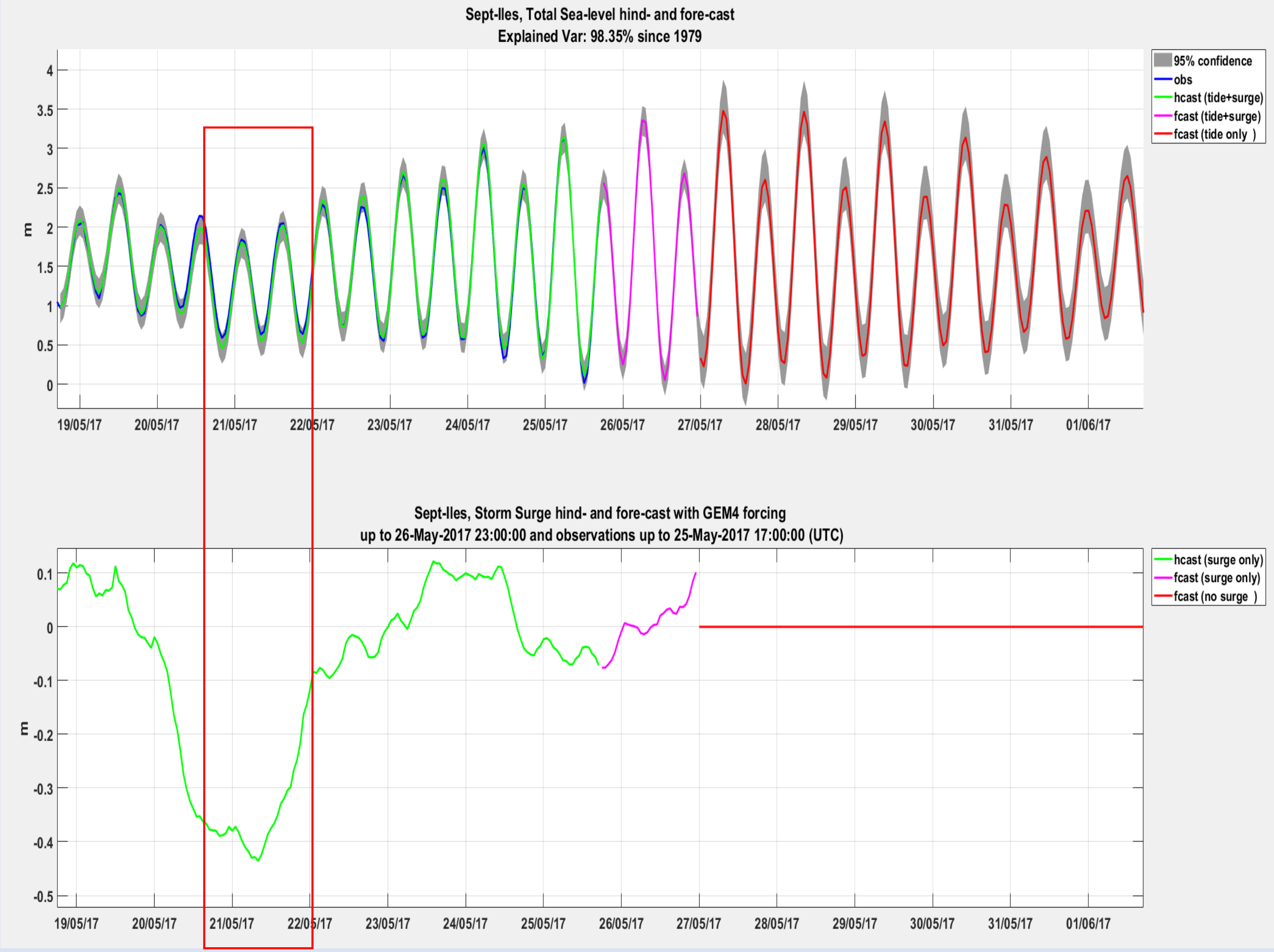
Sept-Iles, Total Sea-level hind- and fore-cast
Explained Var: 98.35% since 1979

An RTG only records up to now.

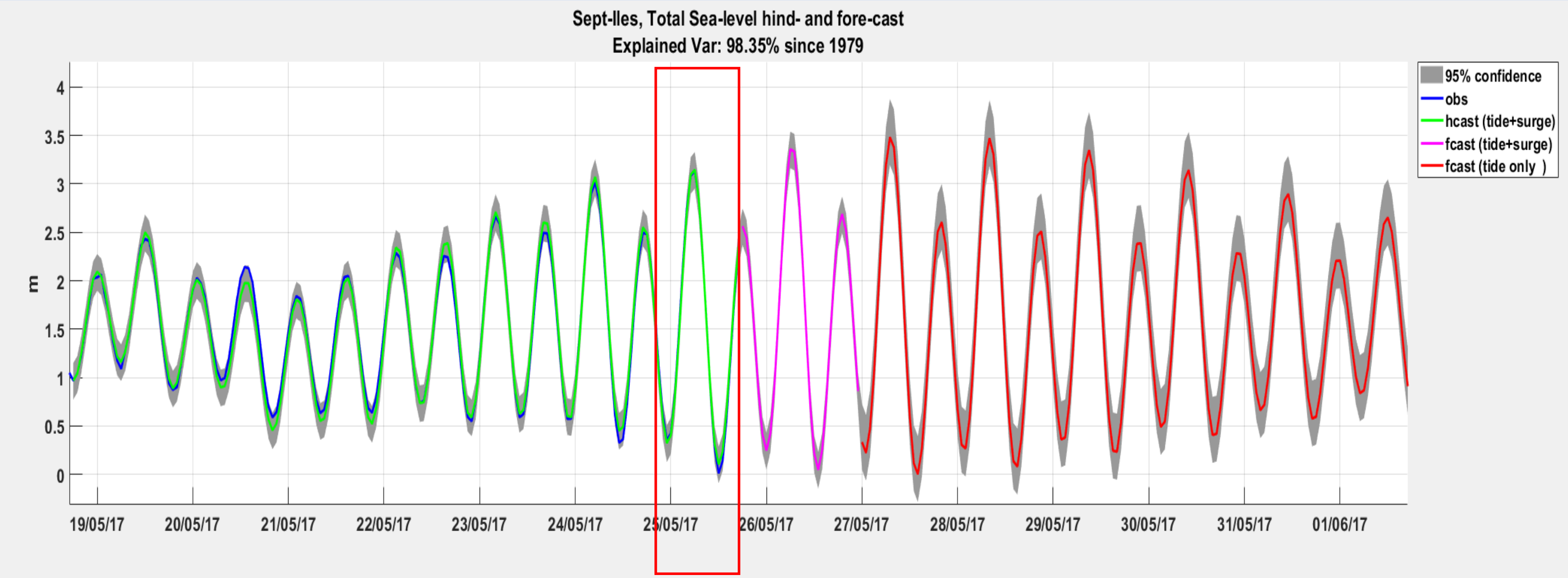


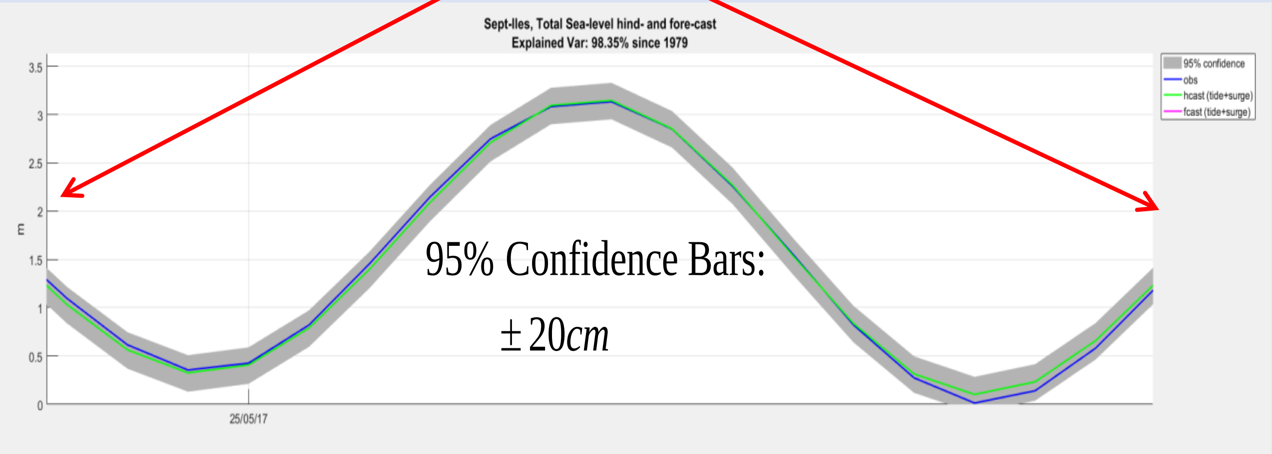
Sept-Iles, Storm Surge hind- and fore-cast with GEM4 forcing
up to 20-Jun-2017 23:00:00 and observations up to 19-Jun-2017 17:00:00 (UTC)

Demo VTG for Sept-Iles running in real-time



Confidence Zone

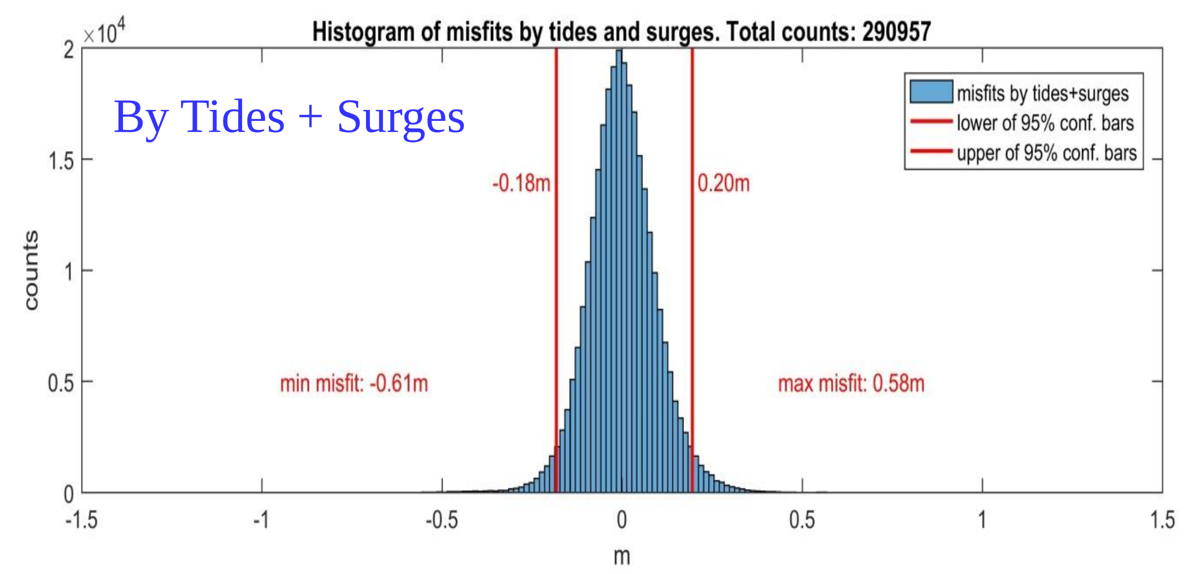




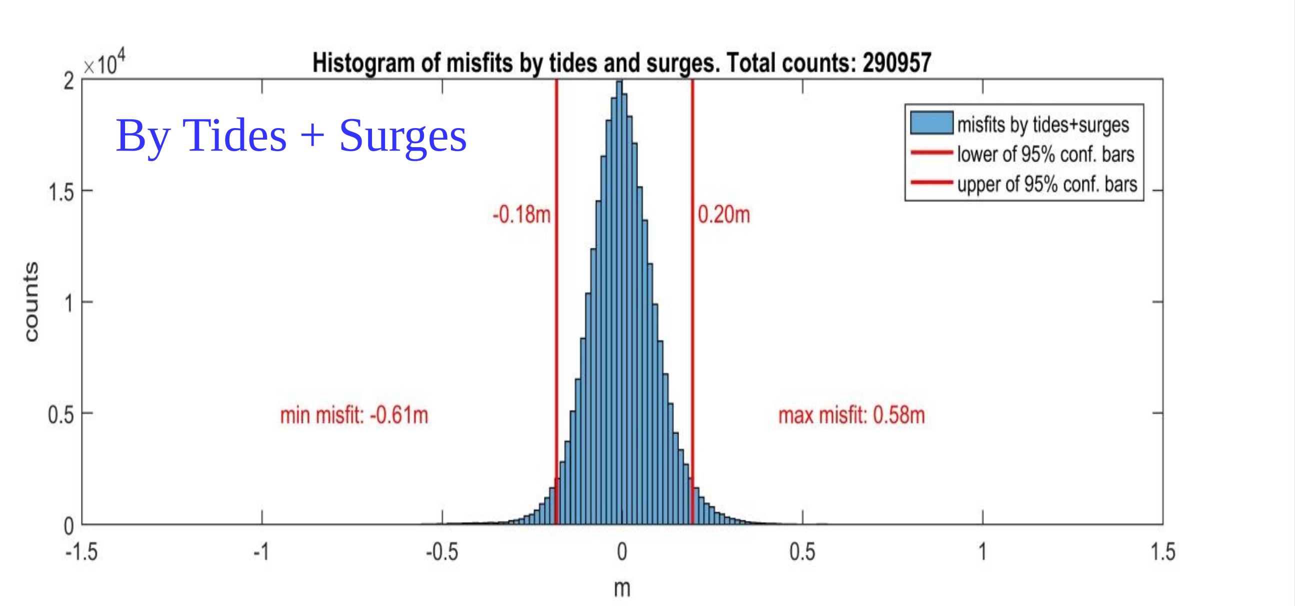
95% Confidence Bars:

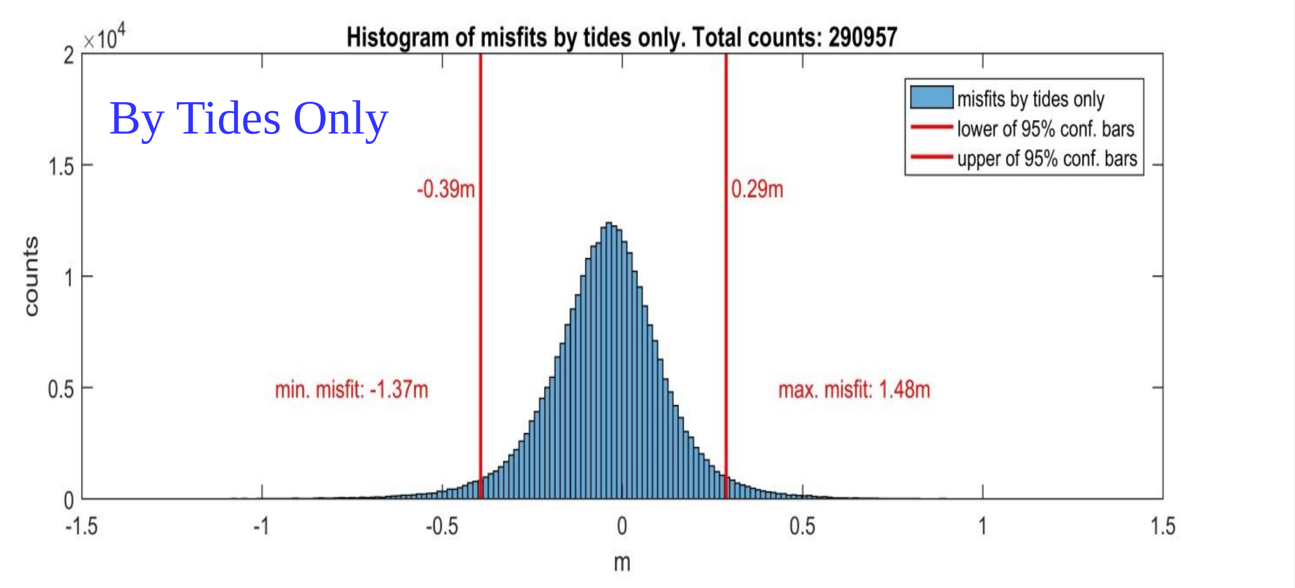
$\pm 20cm$

Stats of Misfits

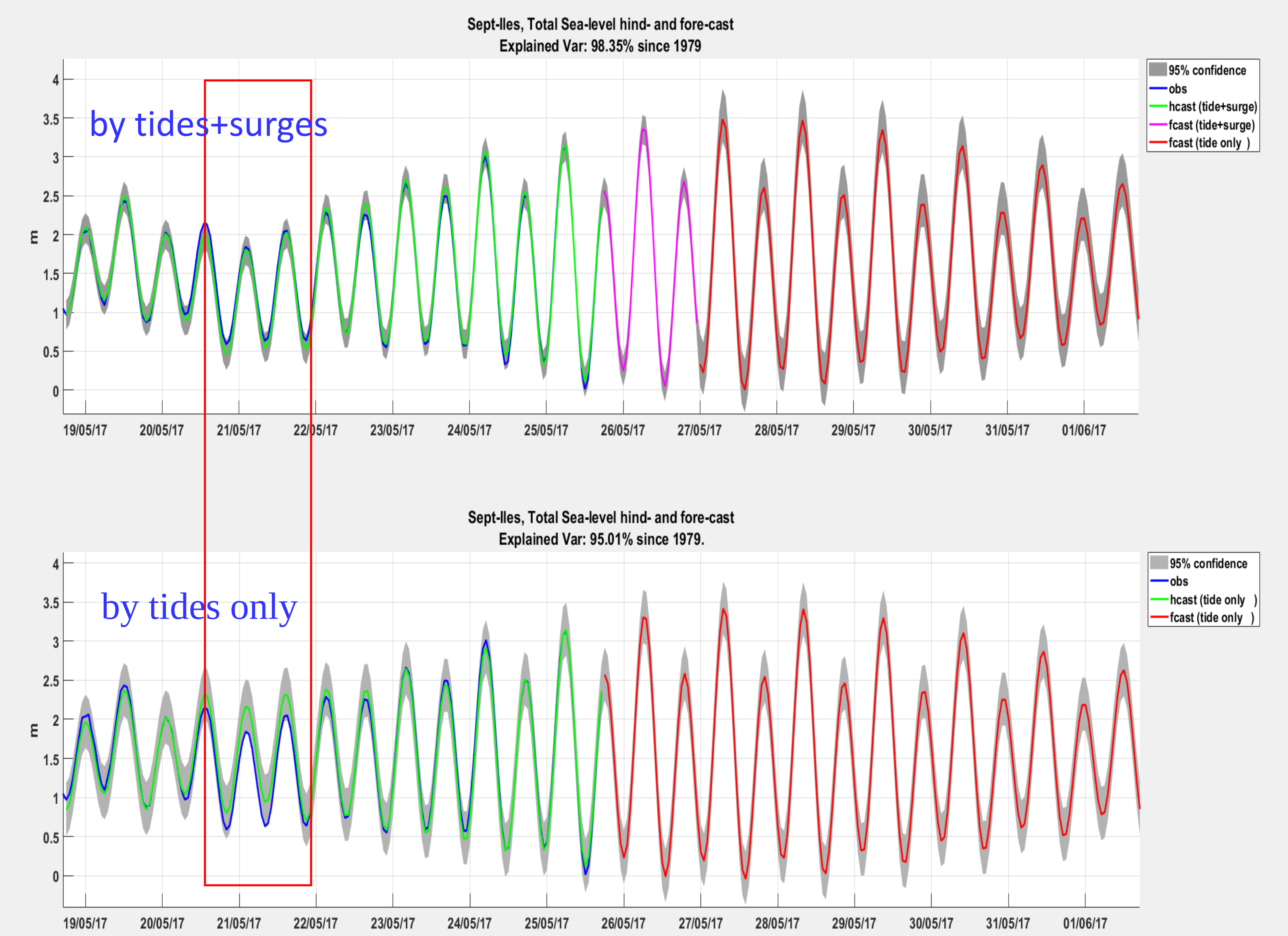


Stats of Misfits

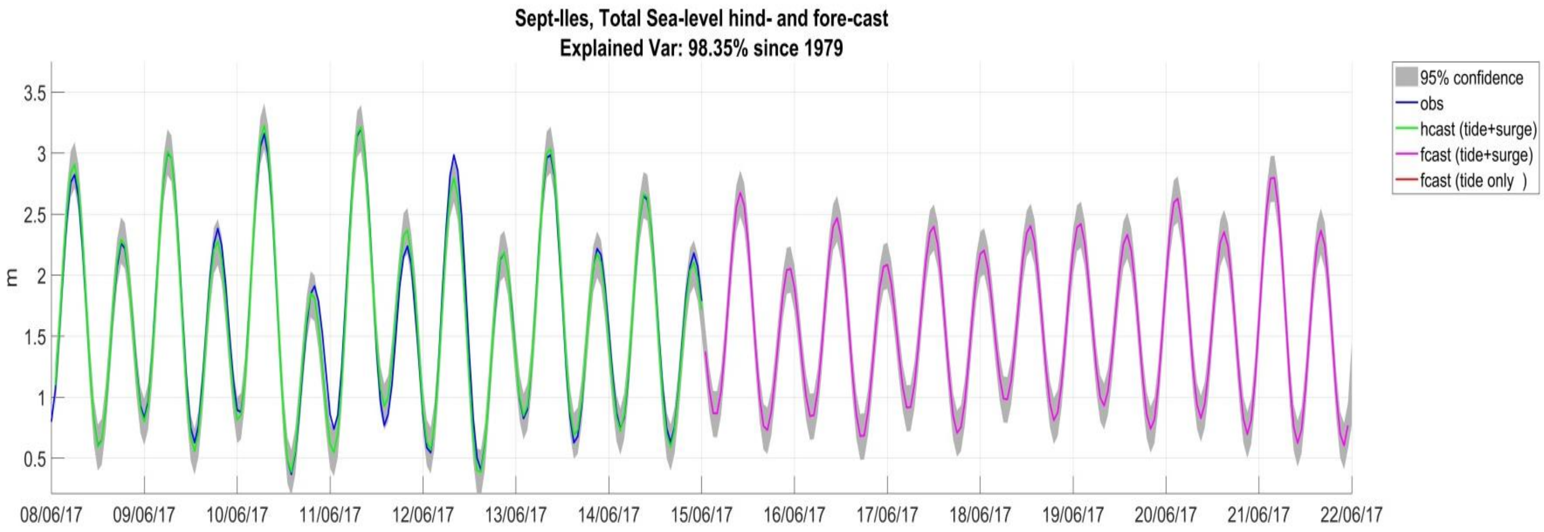


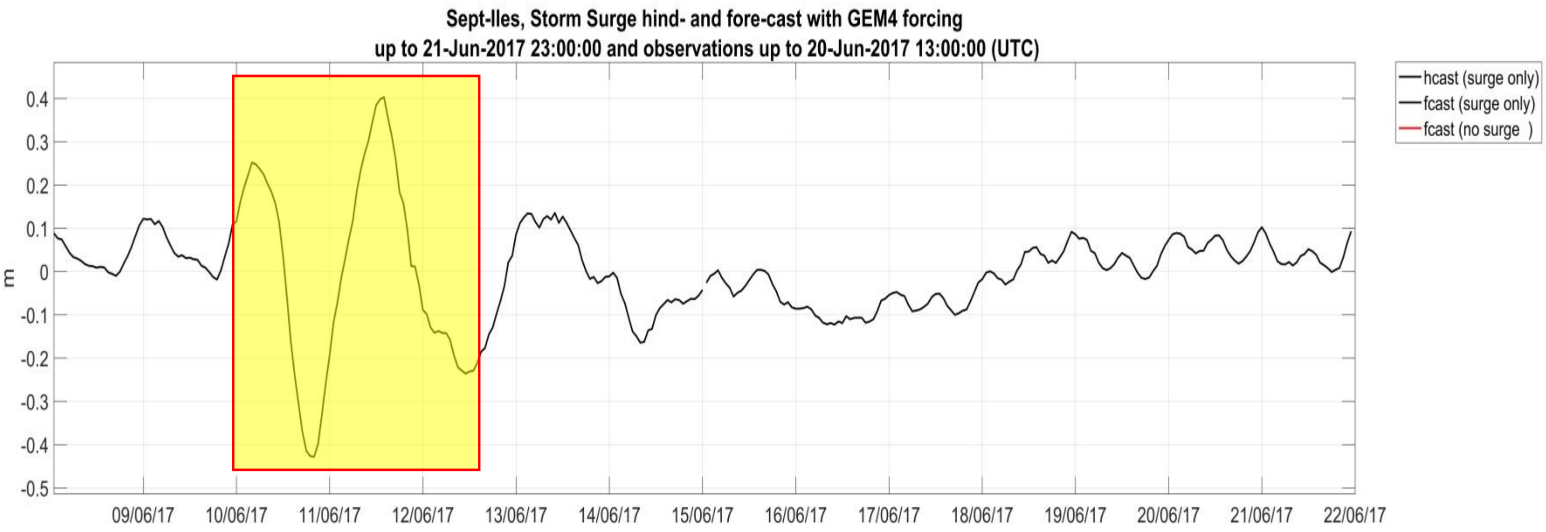


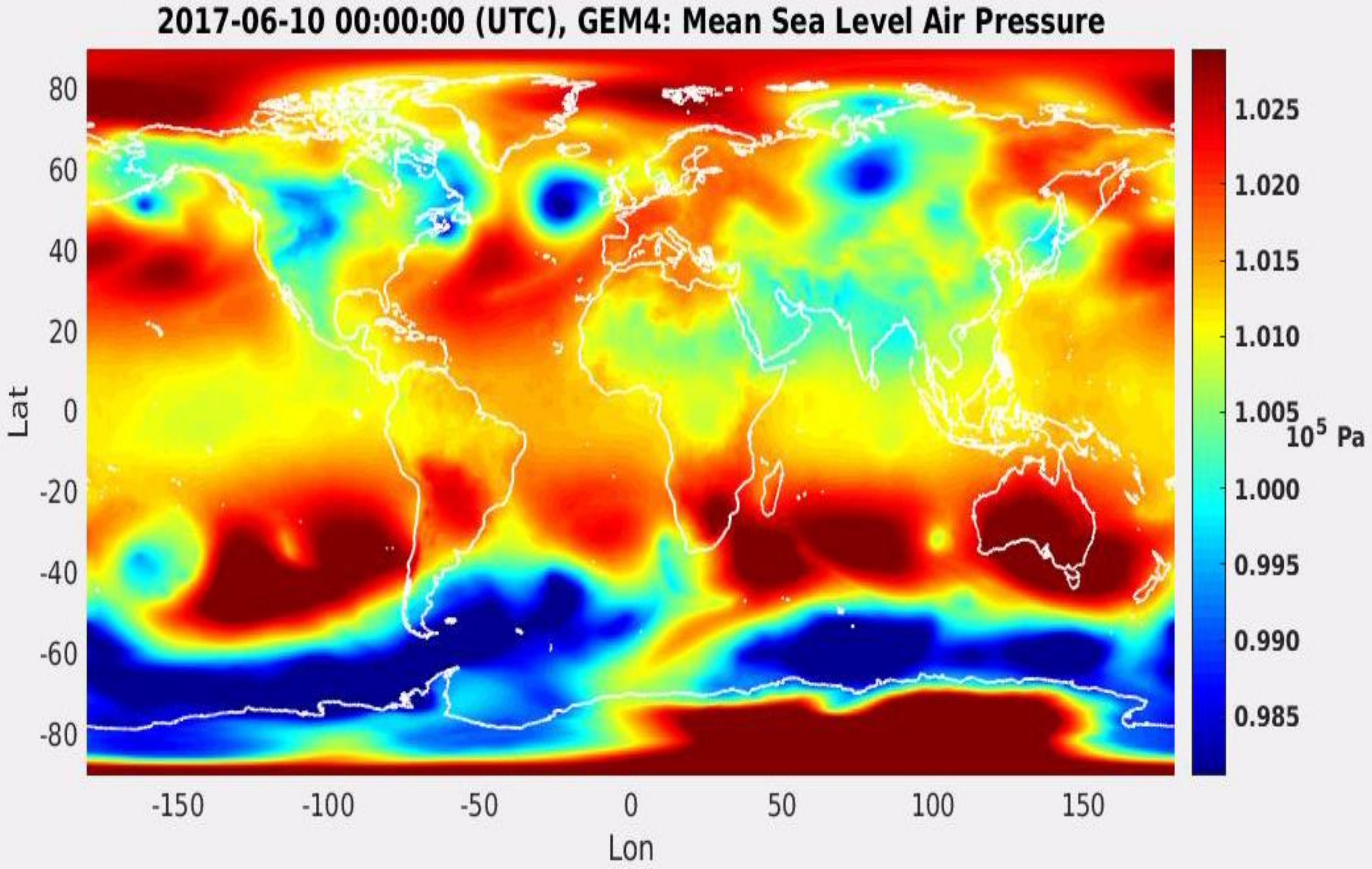
Demo VTG for Sept-Iles running in real-time



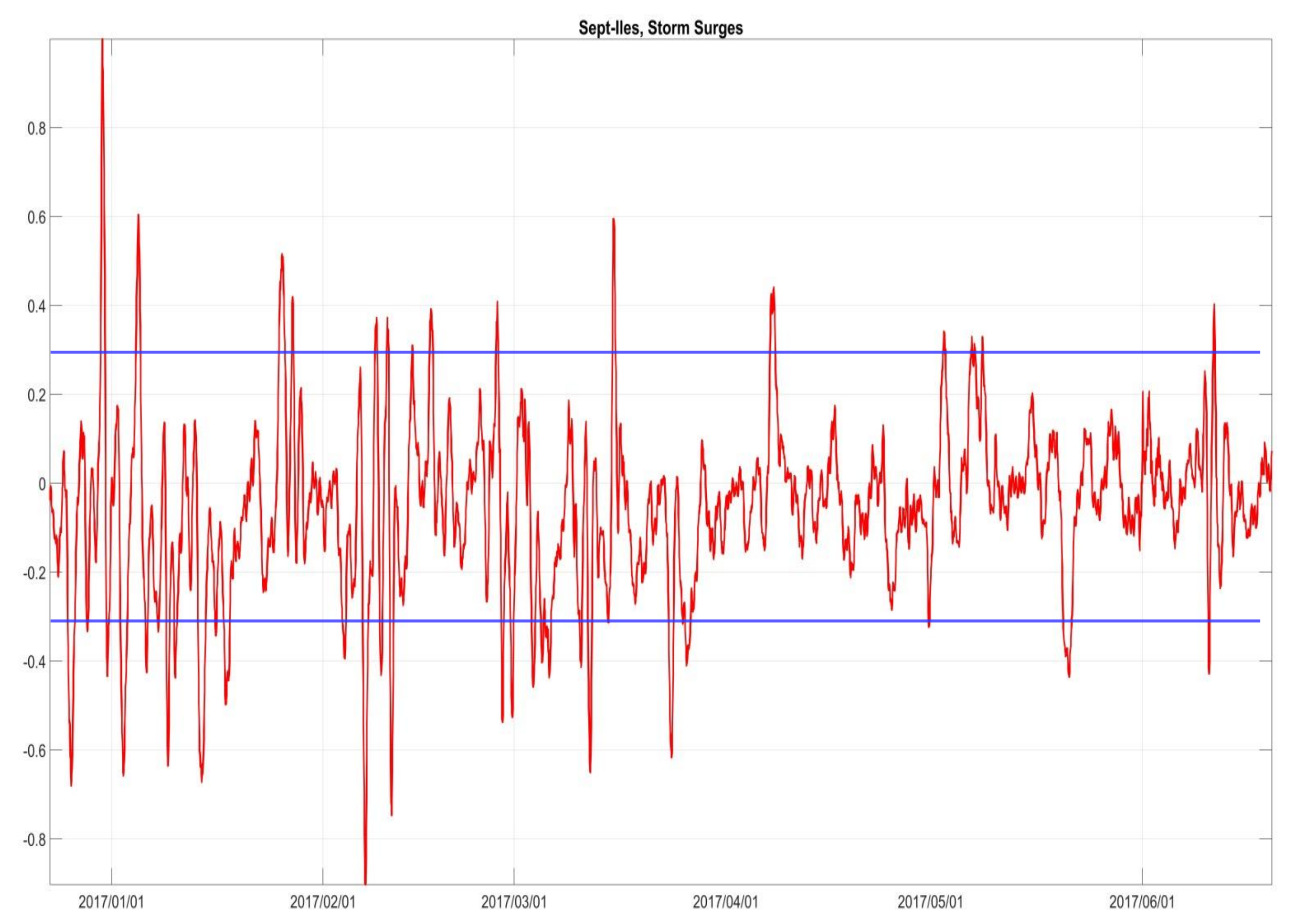
Demo VTG for Sept-Iles running in real-time



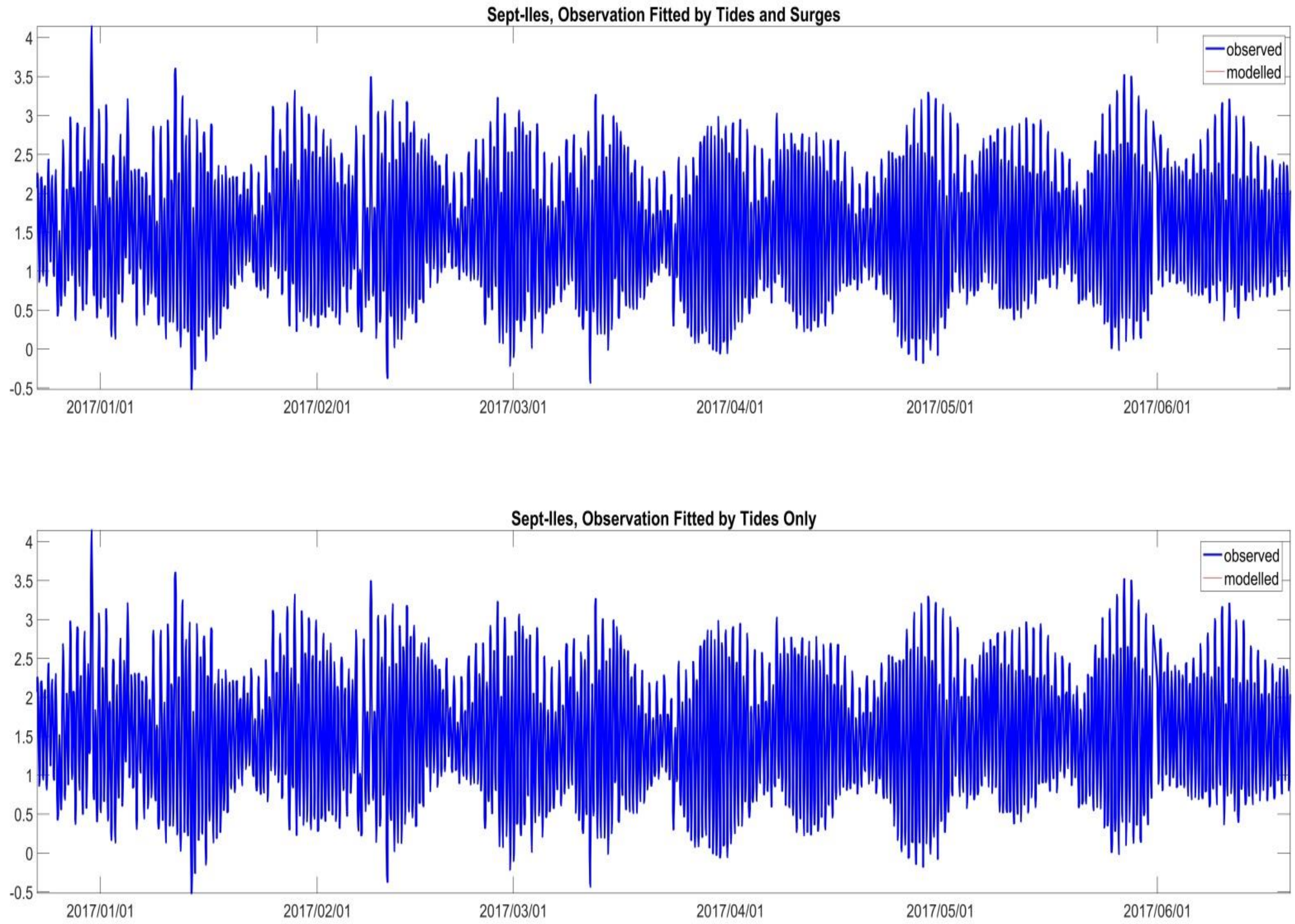



2017-06-10 00:00:00 (UTC), GEM4: Mean Sea Level Air Pressure
Lat
80
60
40
20
0
-20
-40
-60
-80
Lon
-150
-100
-50
0
50
100
150
1.025
1.020
1.015
1.010
1.005
1.000
0.995
0.990
0.985
10^5 Pa

Storm Surges at Sept-Iles, 2016/12/23 to 2017/06/20



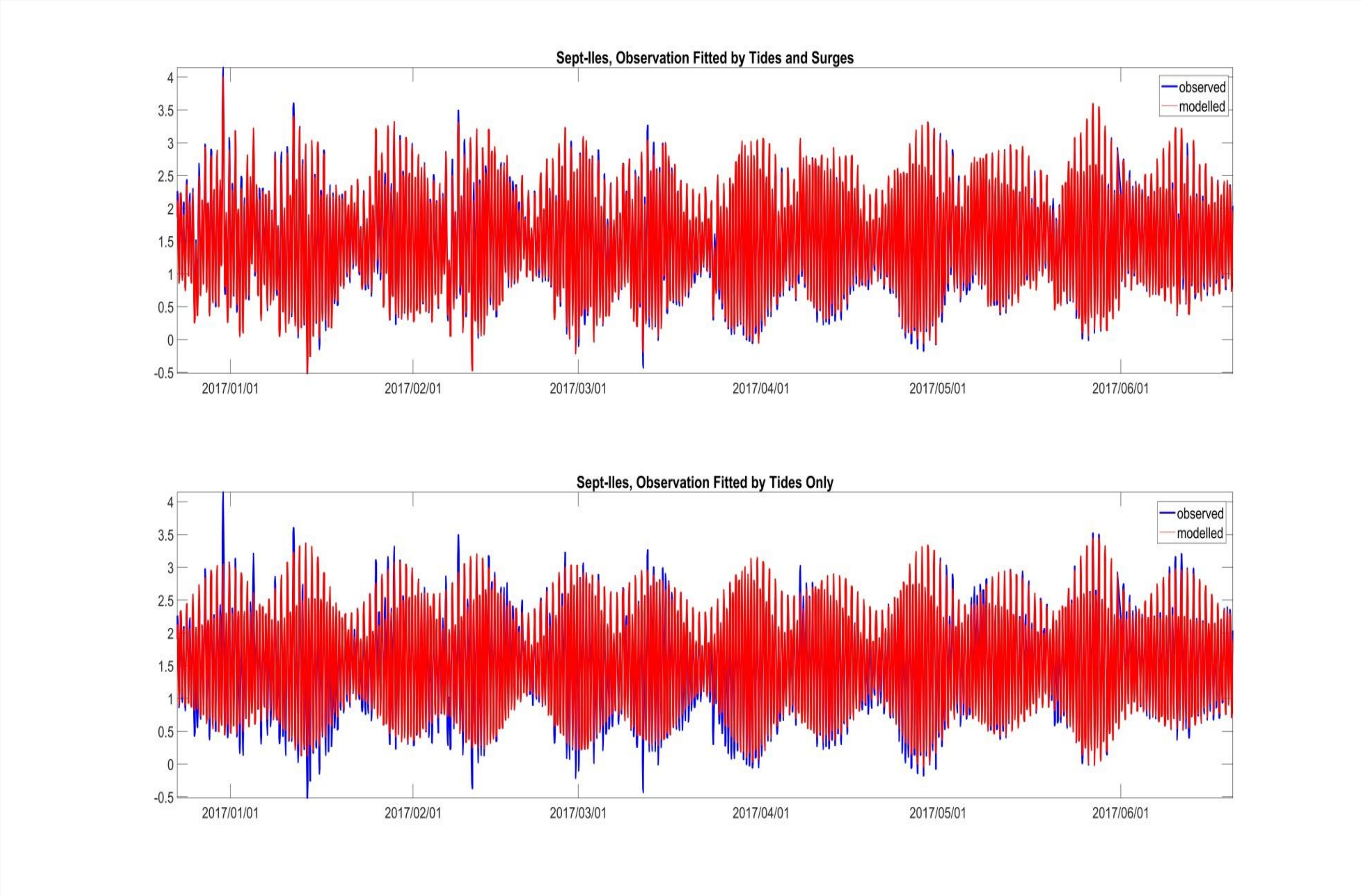
Observations at Sept-Iles, 2016/12/23 to 2017/06/20



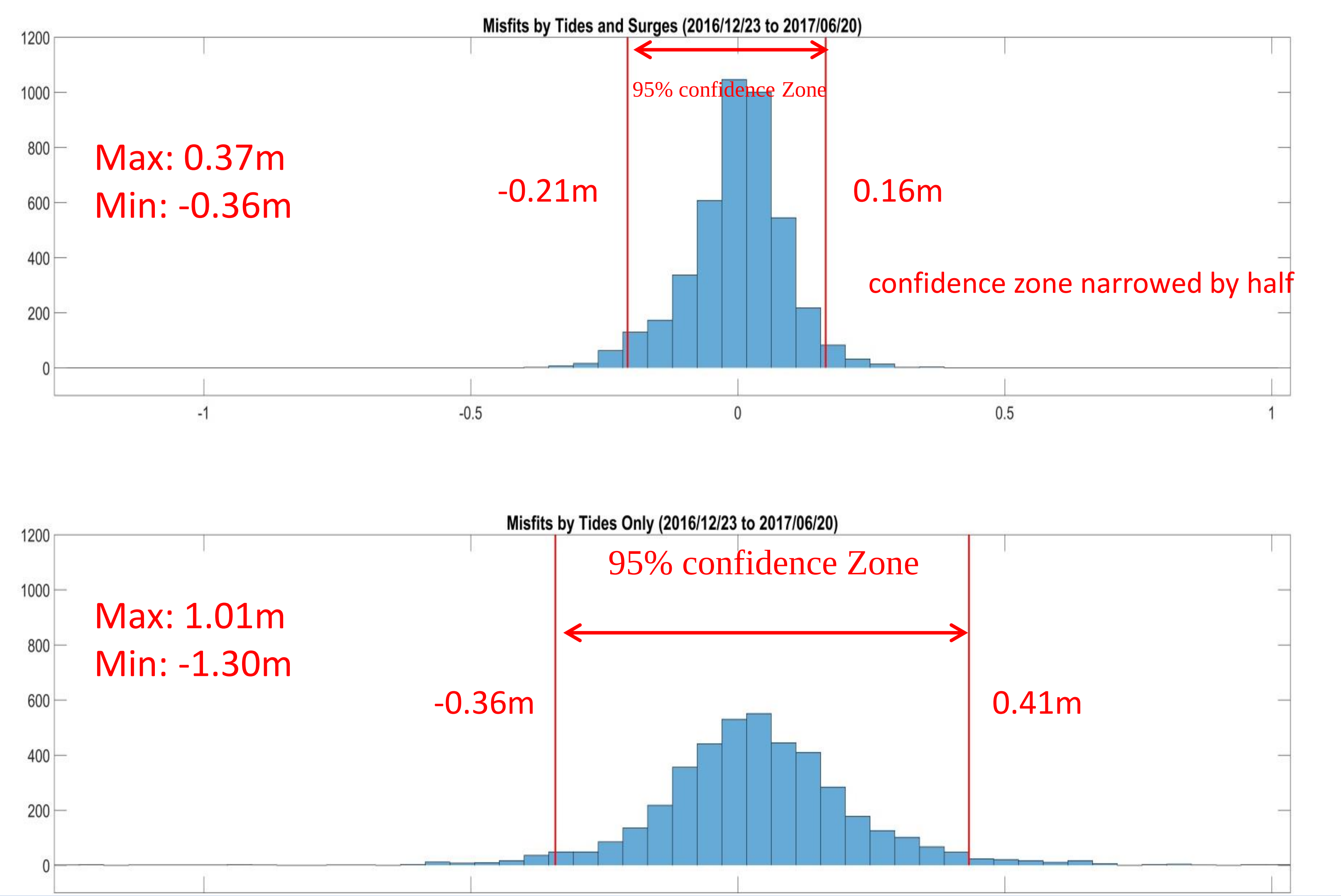
Sept-Iles, Observation Fitted by Tides and Surges

Sept-Iles, Observation Fitted by Tides Only

Observations at Sept-Iles, 2016/12/23 to 2017/06/20



Observations at Sept-Iles, 2016/12/23 to 2017/06/20



95% confidence Zone

Max: 0.37m

Min: -0.36m

-0.21m

0.16m

confidence zone narrowed by half

95% confidence Zone

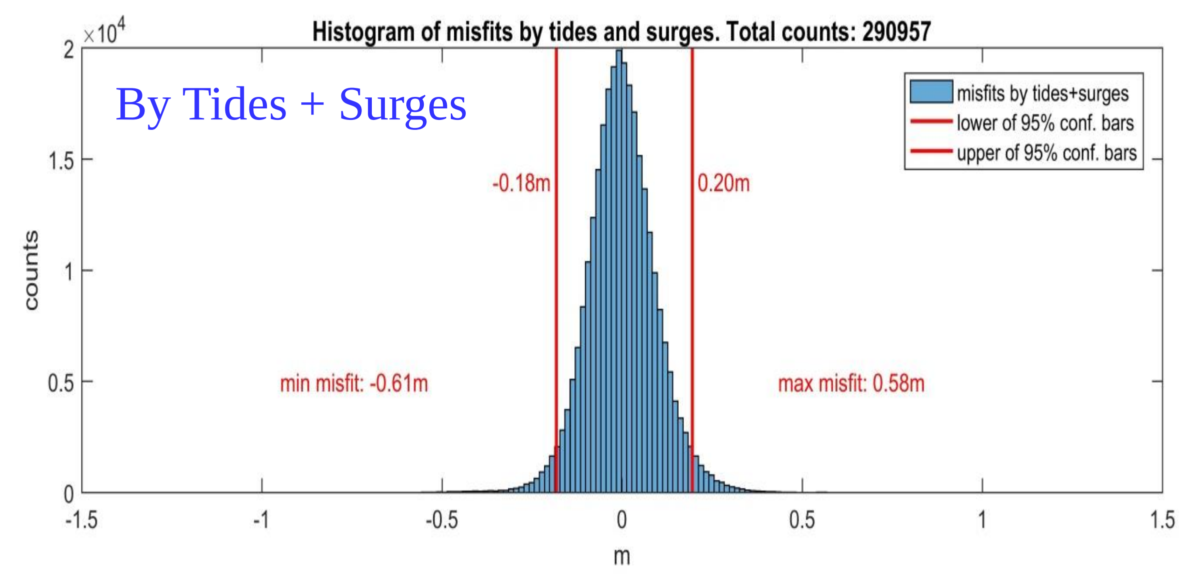
Max: 1.01m

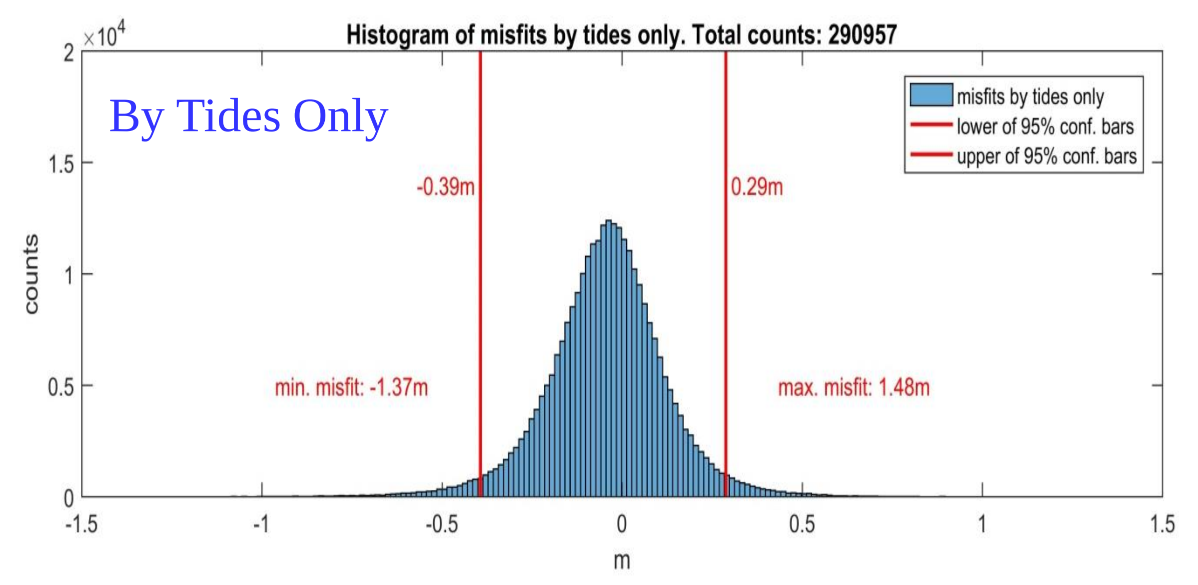
Min: -1.30m

-0.36m

0.41m

Stats of Misfits

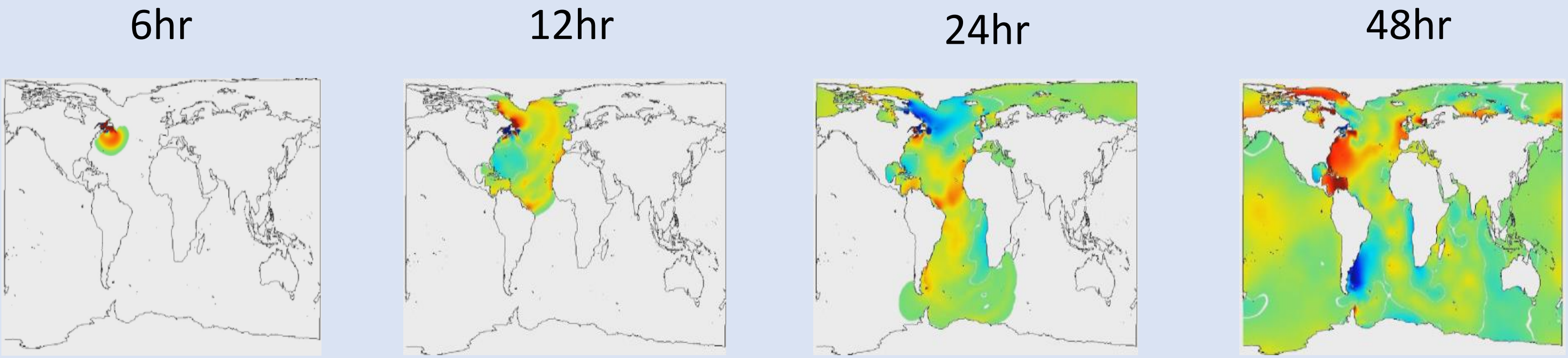




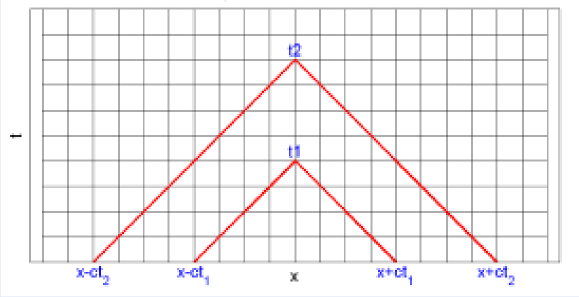
VTG is an Application of ASGF

- ASGF theory (Xu 2007, 2011)
- Application to tsunami problems (Xu 2007, Xu and Song 2013)
- Application to storm surge problems (Xu 2015a,b, Xu et al 2015)
- Application to tide + surge problems, VTG (Xu 2017)

What is an ASGF?

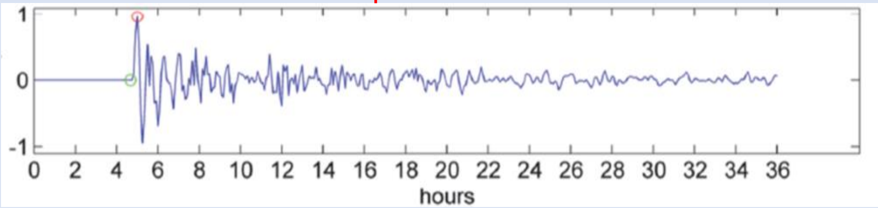


The rows are the domain of dependence.

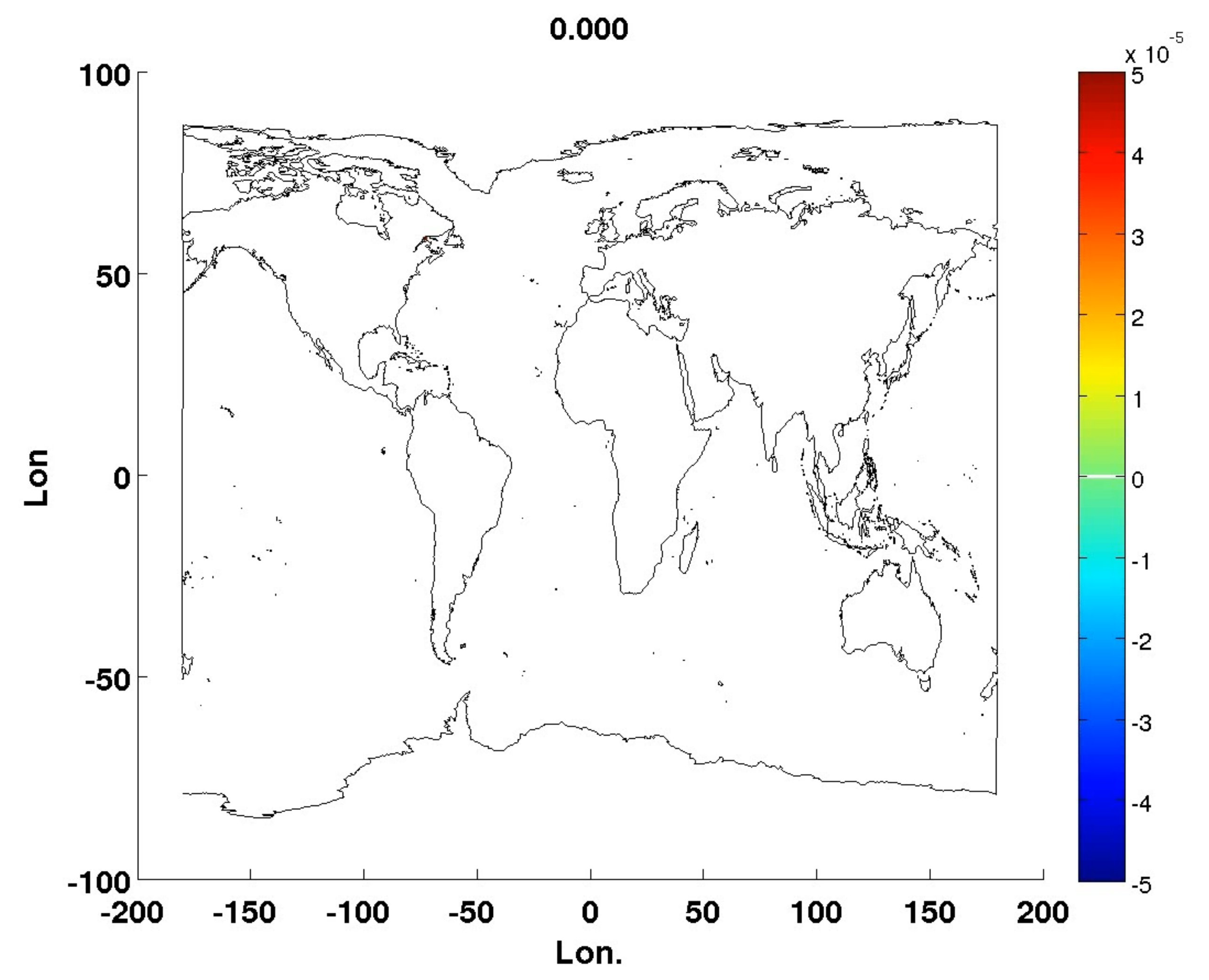


$$\underbrace{\mathbf{G}}_{m\times n}=\begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix}$$

The columns are Green's functions for δ-forcings placed at different grid points.

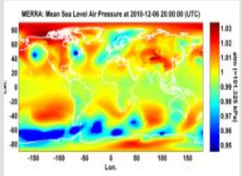


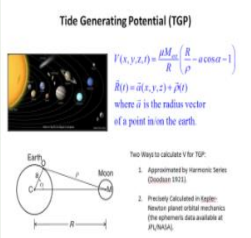
All-Source Green’s Function (ASGF)

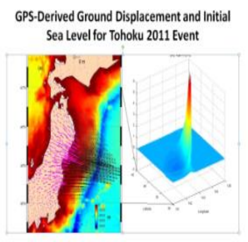


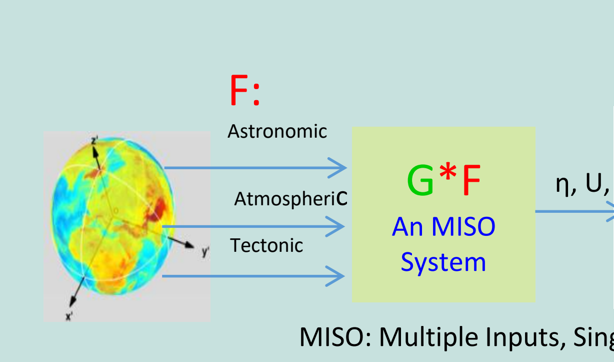
ASGF is also MISO
(Xu, 2007, 2011, 2013, 2015a,b)

ASGF: All-Source Green's Function








F:

Astronomic

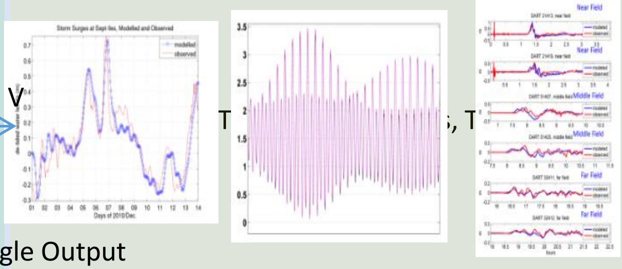
AtmospheriC

Tectonic

G*F

An MISO System

η, U, V



MISO: Multiple Inputs, Single Output

From ASGF to A Regression Model (1)

Xu, 2015a,b Ocean Dynamics

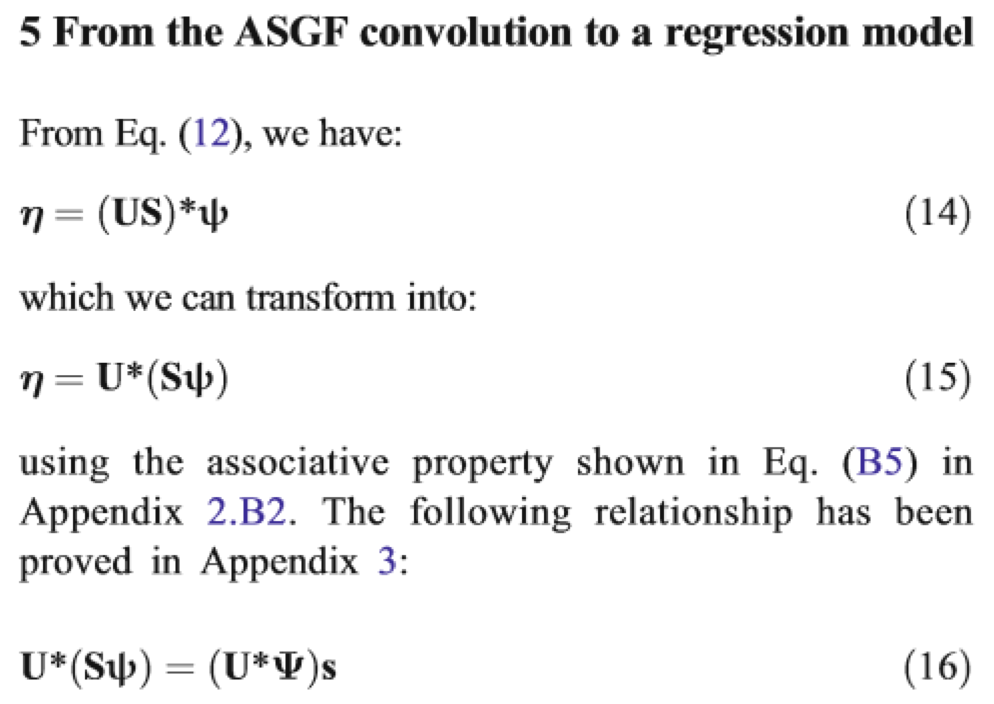
$$\underbrace{\mathbf{G}}_{72\times408550} = \underbrace{\mathbf{U}}_{72\times72}\ \underbrace{\mathbf{S}}_{72\times72}\ \underbrace{\mathbf{V}^T}_{72\times408550}. \quad (9)$$

$$\boldsymbol{\eta} = \left(\mathbf{USV}^T\right)^* \mathbf{f} \quad (10)$$

$$= (\mathbf{US})^* \left(\mathbf{V}^T \mathbf{f}\right) \quad (11)$$

$$= (\mathbf{US})^* \boldsymbol{\psi} \quad (12)$$

An ASGF Regression Model



5 From the ASGF convolution to a regression model

From Eq. (12), we have:

$$\eta = (\mathbf{US})*\psi \tag{14}$$

which we can transform into:

$$\eta = \mathbf{U}*(\mathbf{S}\psi) \tag{15}$$

using the associative property shown in Eq. (B5) in Appendix 2.B2. The following relationship has been proved in Appendix 3:

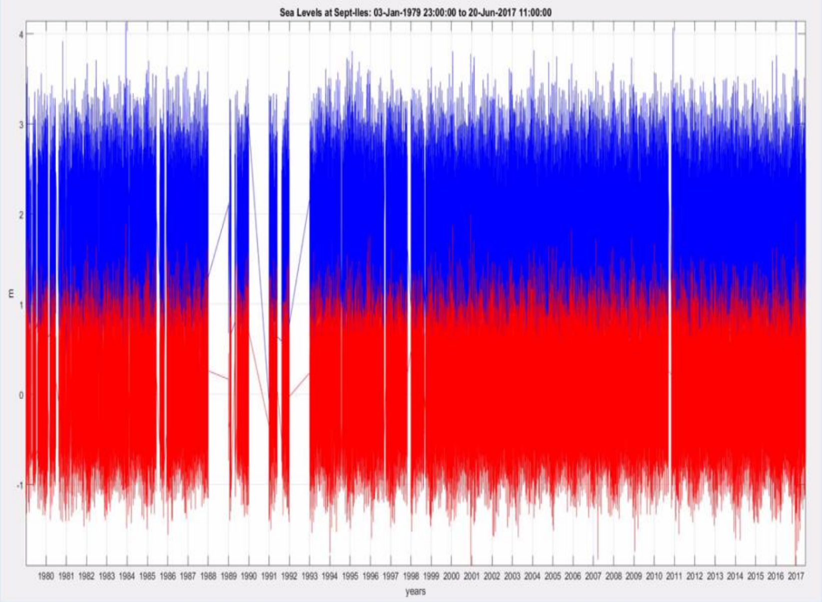
$$\mathbf{U}*(\mathbf{S}\psi) = (\mathbf{U}*\Psi)\mathbf{s} \tag{16}$$

Xu, 2015a,b Ocean Dynamics

$$\mathbf{C} = \mathbf{U}*\Psi \tag{18}$$

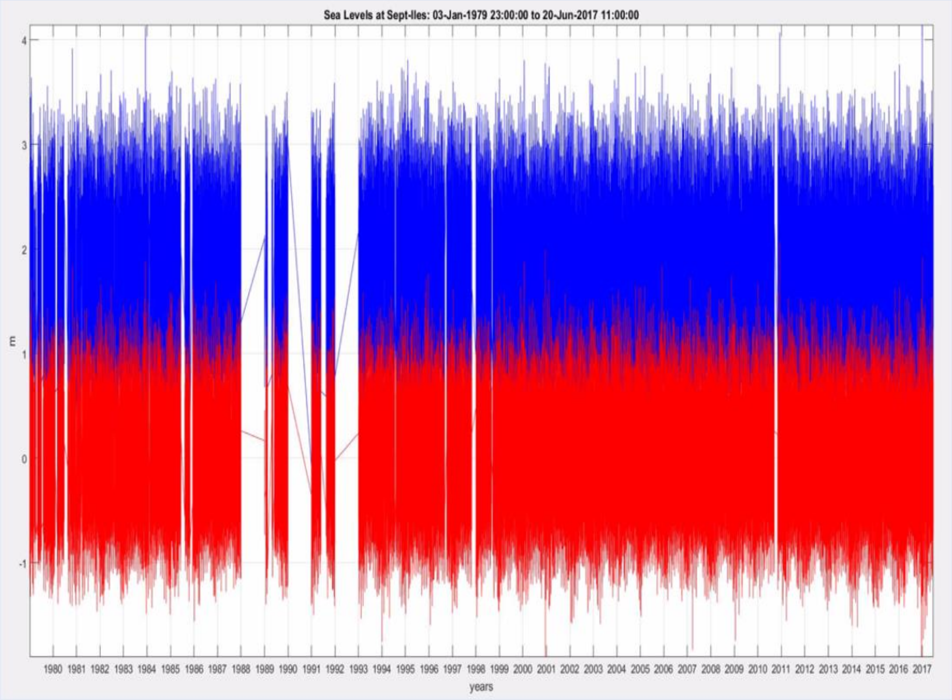
$$\boxed{\eta = \mathbf{C}\ \mathbf{s} + \varepsilon} \longleftarrow \text{ASGF Regression Model} \tag{19}$$

Data Assimilation (Mode Training)

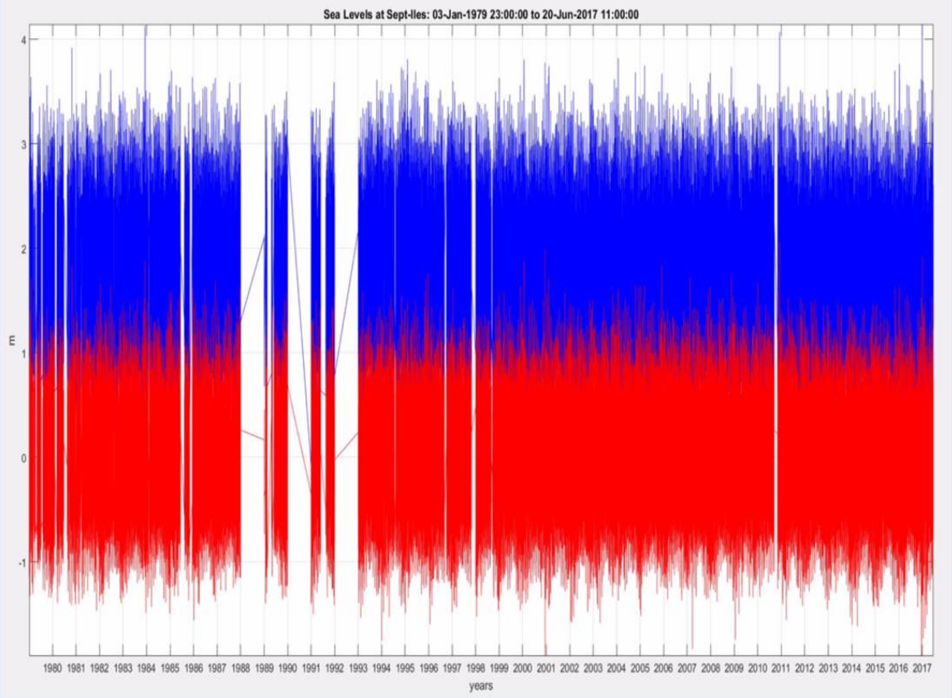


$$\eta_{obs} = \begin{bmatrix} \mathbf{C}_{\mathbf{linear}} & \mathbf{C}_{\mathbf{tides}} & \mathbf{C}_{\mathbf{surges}} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

$$\eta = \begin{bmatrix} \mathbf{C}_{\mathbf{linear}} & \mathbf{C}_{\mathbf{tides}} & \mathbf{C}_{\mathbf{surges}} \end{bmatrix} \begin{bmatrix} 0 \\ S_{2,\mathrm{mod}} \\ S_{3,\mathrm{mod}} \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{0} \\ S_{2,\mathrm{mod}} \\ S_{3,\mathrm{mod}} \\ \eta_{obs} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \\ \mathbf{C}_{\mathbf{linear}} & \mathbf{C}_{\mathbf{tides}} & \mathbf{C}_{\mathbf{surges}} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}$$



$$\left[\begin{array}{c} \mathbf{0} \\ S_{2,\mathrm{mod}} \\ S_{3,\mathrm{mod}} \\ \hline \eta_{obs} \end{array}\right] = \left[\begin{array}{ccc} \mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \\ \hline \mathbf{C}_{\mathbf{linear}} & \mathbf{C}_{\mathbf{tides}} & \mathbf{C}_{\mathbf{surges}} \end{array}\right] \left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array}\right]$$

- Best Fit between the Model and the Observation.
- Never Fails!
- Tides, Surges and Linear Trends are assimilated simultaneously.

Number of Mode Parameters

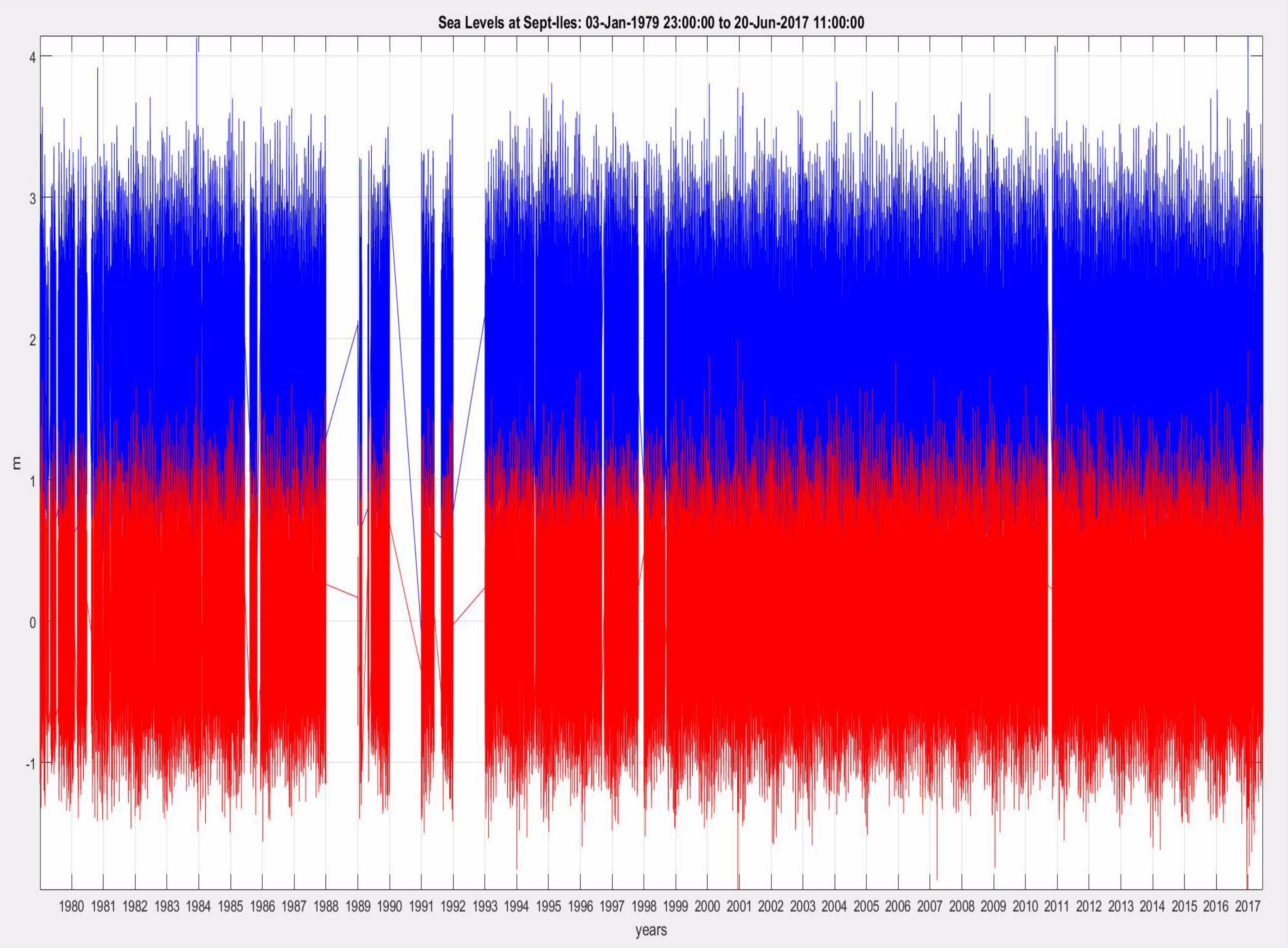
$$\begin{array}{r} \text{2 elements} \\ \text{72 elements} \\ \text{72 elements} \\ \text{N elements,} \\ (\text{N} \gg 1) \end{array} \begin{bmatrix} \mathbf{0} \\ S_{2,\text{mod}} \\ S_{3,\text{mod}} \\ \eta_{obs} \end{bmatrix} = \begin{array}{c} \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \\ \mathbf{C}_{\textbf{linear}} & \mathbf{C}_{\textbf{tides}} & \mathbf{C}_{\textbf{surges}} \end{bmatrix} \\ \begin{matrix} \text{2 colm} & \text{72 colm} & \text{72 colm} \end{matrix} \end{array} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \begin{array}{l} \text{2 elements} \\ \text{72 elements} \\ \text{72 elements} \end{array}$$

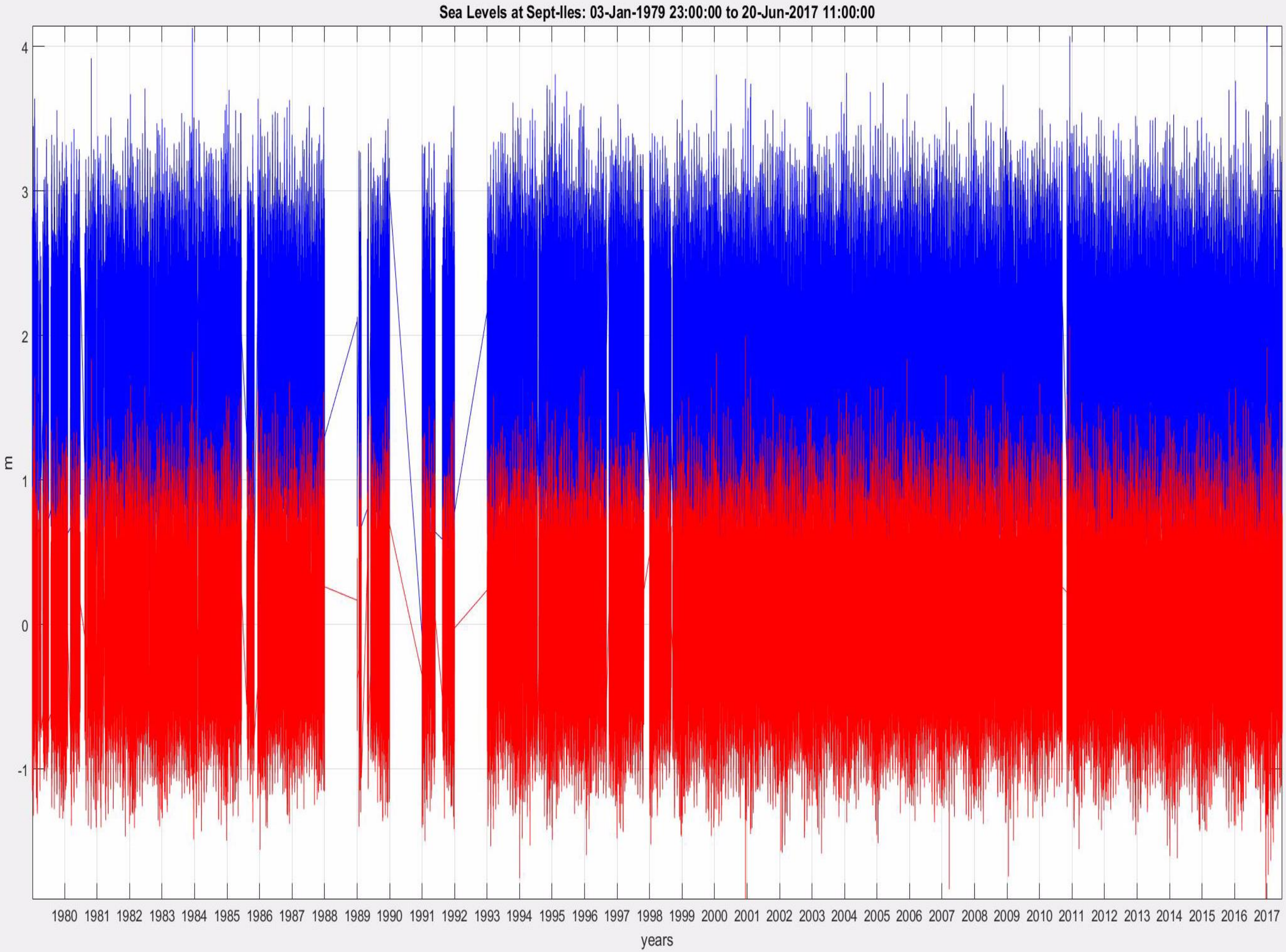
```
K>> tic
        I=eye(numel(x0));
        H=[I; A]; d=[x0;b];
        P1=(H'*H)\I;
        x1=P1*(H'*d);
        y1=A*x1;
        toc
Elapsed time: 0.389253 s.
```

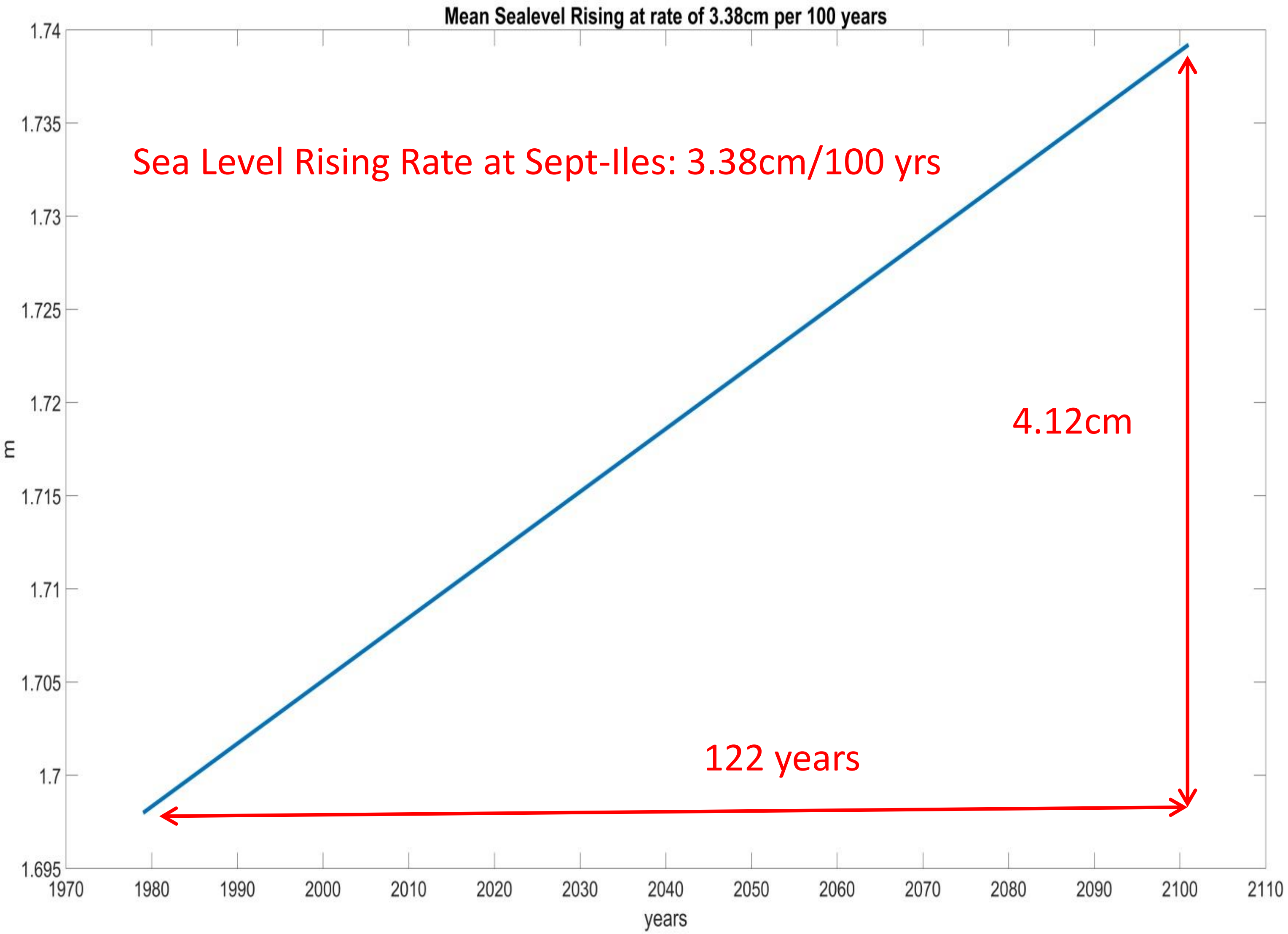
Size of H: 292,632x146
(38 years of hourly data)

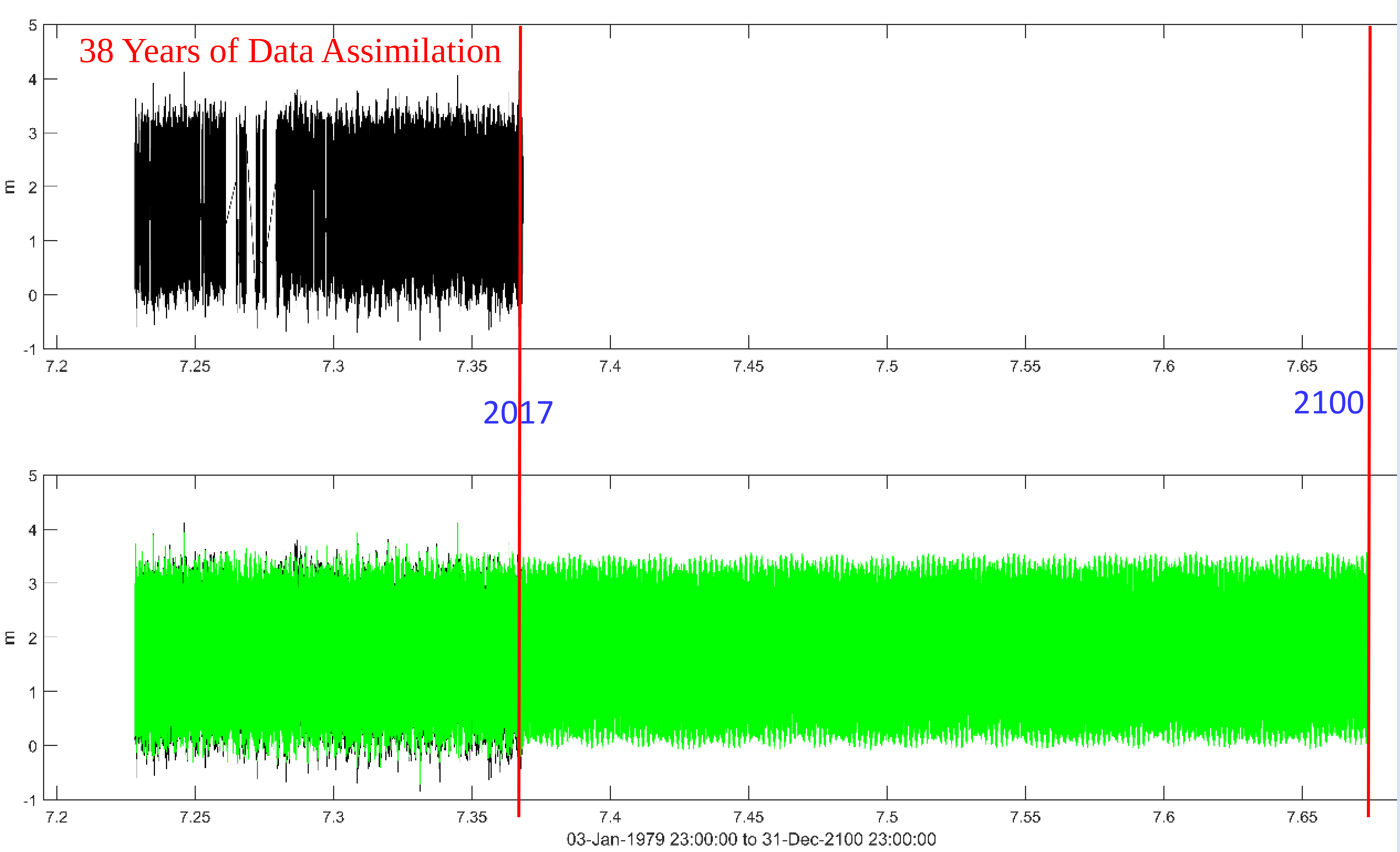
Why it works so fast and accurately?

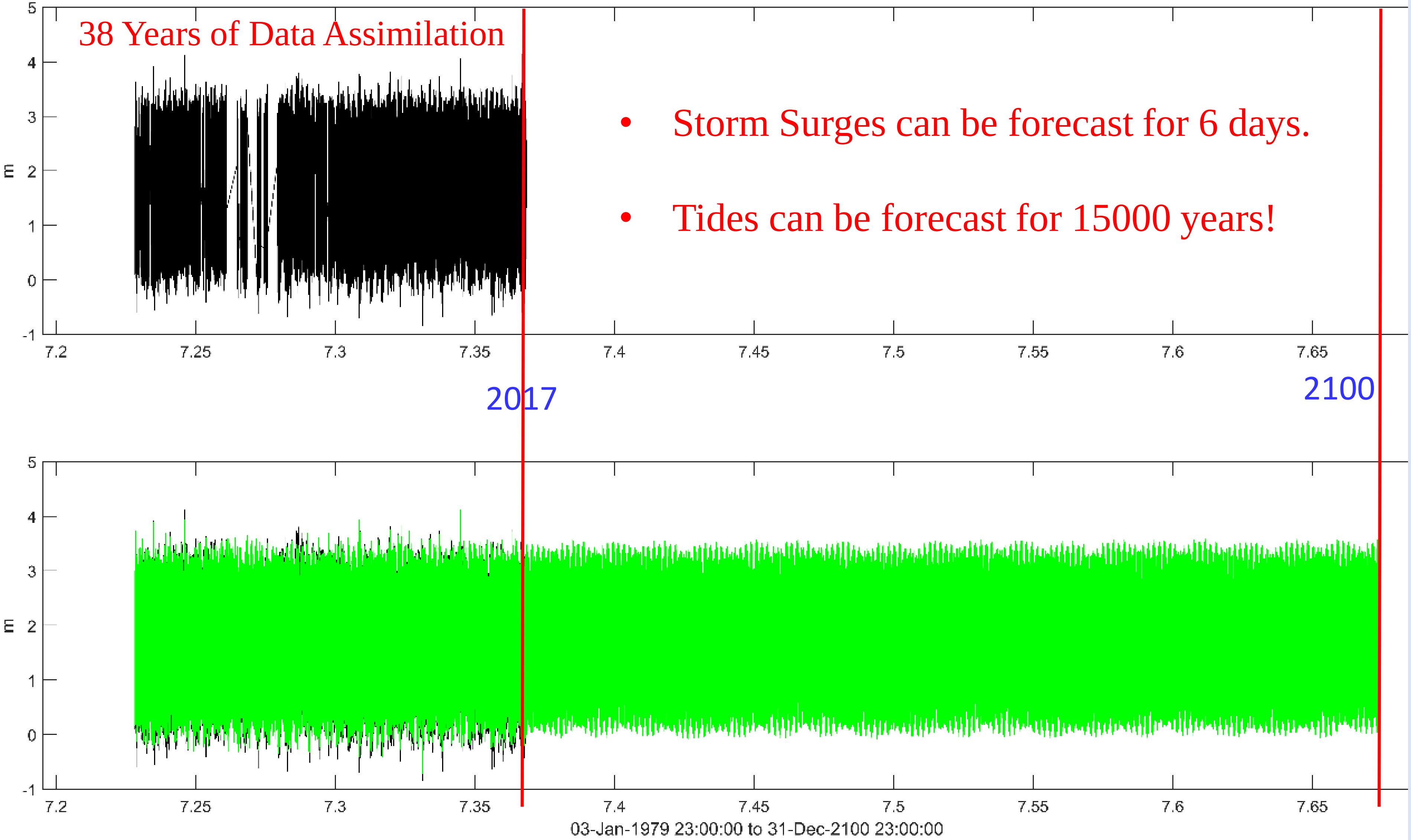
- All-Source Green’s Function (ASGF)
- ASGF Regression Model


Sea Levels at Sept-Iles: 03-Jan-1979 23:00:00 to 20-Jun-2017 11:00:00
m
4
3
2
1
0
-1
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017
years


Sea Levels at Sept-Iles: 03-Jan-1979 23:00:00 to 20-Jun-2017 11:00:00
m
4
3
2
1
0
-1
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017
years

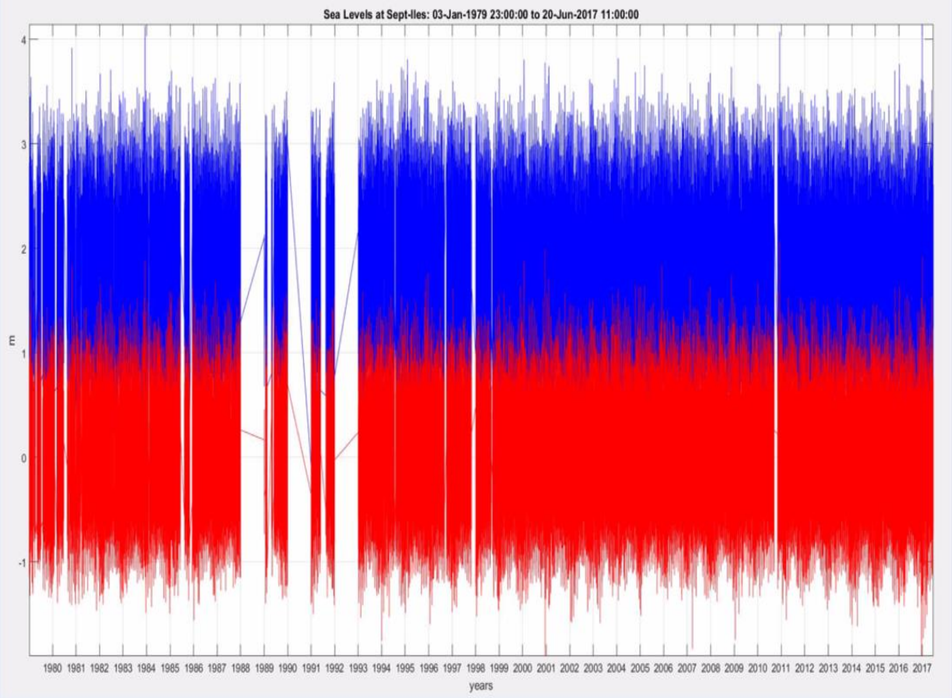

Mean Sealevel Rising at rate of 3.38cm per 100 years
1.74
1.735
1.73
1.725
1.72
1.715
1.71
1.705
1.7
1.695
m
1970
1980
1990
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
2110
years
Sea Level Rising Rate at Sept-Iles: 3.38cm/100 yrs
4.12cm
122 years


38 Years of Data Assimilation
2017
2100
m
5
4
3
2
1
0
-1
7.2
7.25
7.3
7.35
7.4
7.45
7.5
7.55
7.6
7.65
03-Jan-1979 23:00:00 to 31-Dec-2100 23:00:00



38 Years of Data Assimilation

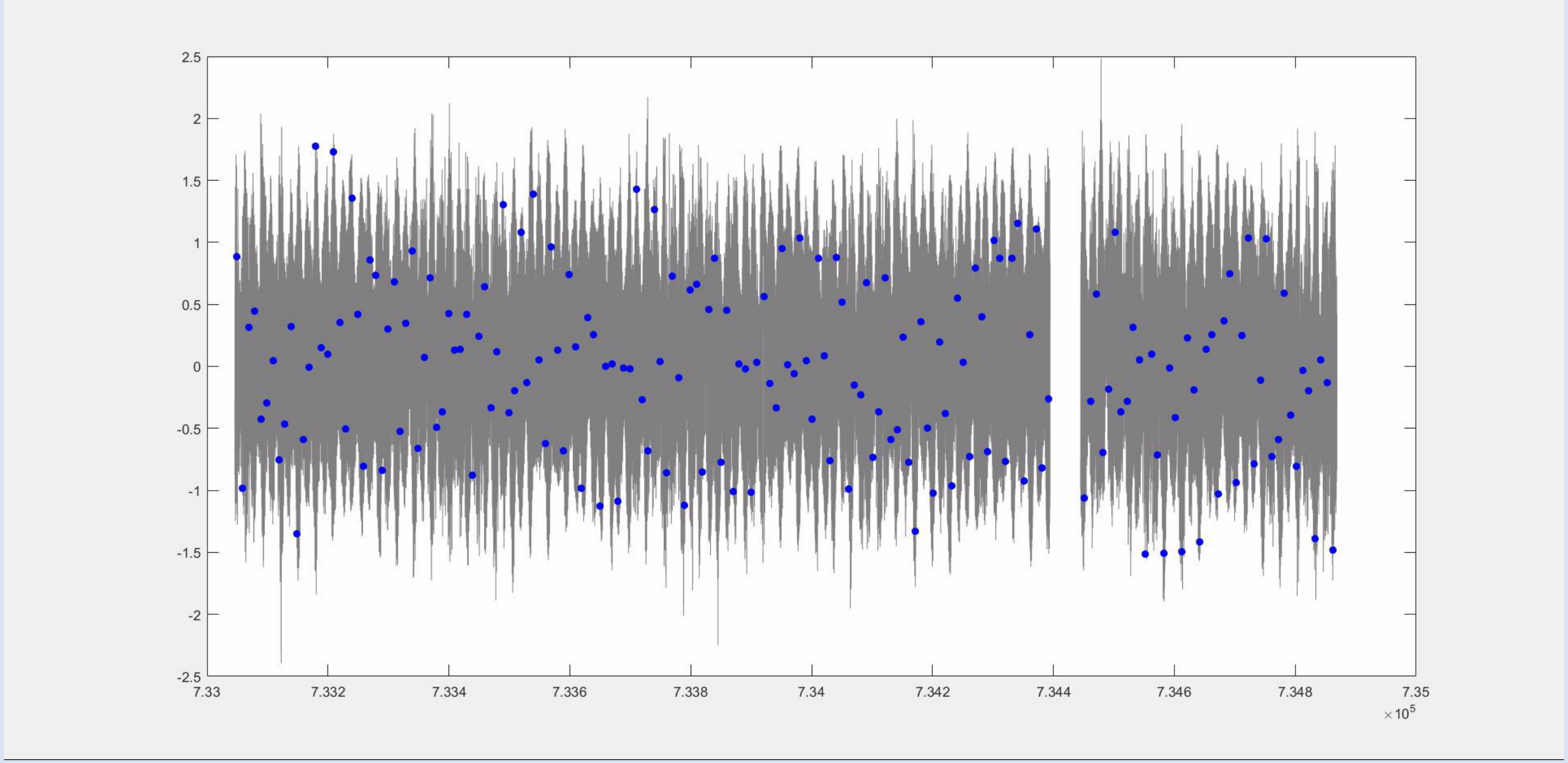
- Storm Surges can be forecast for 6 days.
- Tides can be forecast for 15000 years!



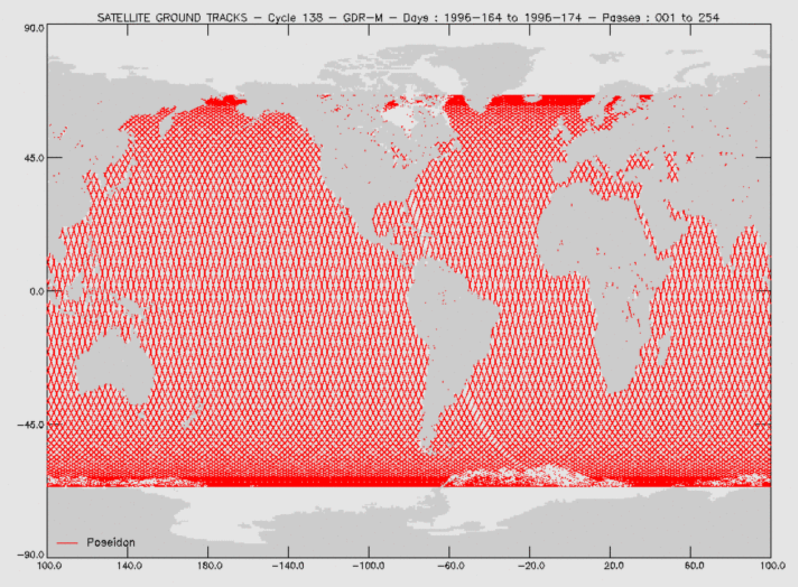
$$\left[\begin{array}{c} \mathbf{0} \\ S_{2,\mathrm{mod}} \\ S_{3,\mathrm{mod}} \\ \hline \eta_{obs} \end{array}\right] = \left[\begin{array}{ccc} \mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \\ \hline \mathbf{C}_{\mathbf{linear}} & \mathbf{C}_{\mathbf{tides}} & \mathbf{C}_{\mathbf{surges}} \end{array}\right] \left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array}\right]$$

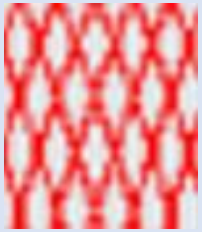
- Best Fit between the Model and the Observation.
- Never Fails!
- Tides, Surges and Linear Trends are assimilated simultaneously.

Recursive Regression using one data every ten days



De-tide Satellite Sea Surface Topography Data





- Harmonic Analyses Method (Doodson, 1921), challenged by large track repeating cycle (9.9156 days, compared to 0.5 or 1 day of tidal periods, and to Nyquuist sampling rate, 0.25 days).

- Local Green's Function Method (Response Method, Munk and Cartwright, 1966), challenged by a single source point (only local forcing).

- All-Source Green's Function Method (ASGF; Xu, 2016)

Development of Tidal Data Analyses

- Darwin (1897), Doodson's harmonic method (1921)
- Munk and Cartwright's Local Green Function Method (1966)
- Xu's Global All-Source Green Function Method (2016, 2017)

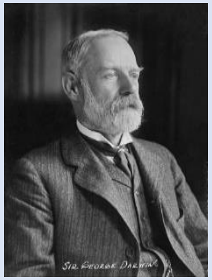
Frequency-based approach

$$V=\sum_{n=1}^{N} A_n \cos(\omega_n t+\phi_n)$$

where

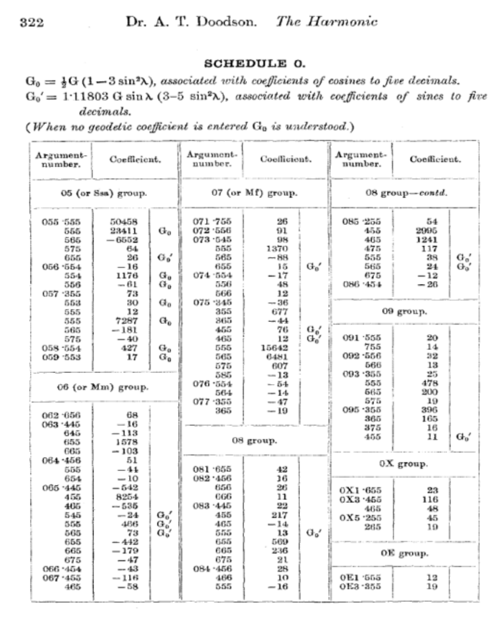
$$N=\begin{cases}\infty, & \text{in theory}\\ 386, & \text{in practice (error, \textasciitilde mm)}\end{cases}$$

Darwin 1897



Doodson 1921

Long term



322 Dr. A. T. Doodson. *The Harmonic*

SCHEDULE 0.

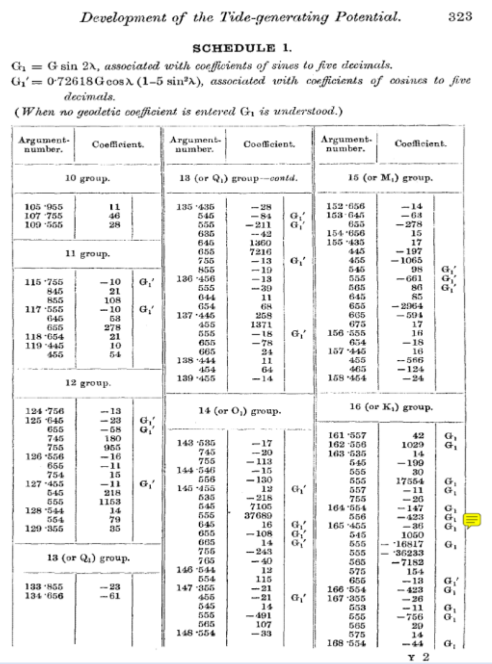
$G_0 = \frac{1}{2}G(1-3\sin^2\lambda)$, *associated with coefficients of cosines to five decimals.*
$G_0' = 1{\cdot}11803\,G\sin\lambda\,(3-5\sin^2\lambda)$, *associated with coefficients of sines to five decimals.*
(*When no geodetic coefficient is entered* G_0 *is understood.*)

Argument-number.	Coefficient.	
05 (or Ssa) group.		
055 ·555	50458	
555	23411	G_0
565	−6552	
575	64	
655	26	G_0'
056 ·554	−16	
554	1176	G_0
556	−61	G_0
057 ·355	73	
553	30	G_0
555	12	
555	7287	G_0
565	−181	
575	−40	
058 ·554	427	G_0
059 ·553	17	G_0
06 (or Mm) group.		
062 ·656	68	
063 ·445	−16	
645	−113	
655	1578	
665	−103	
064 ·456	51	
555	−44	
654	−10	
065 ·445	−542	
455	8254	
465	−535	
545	−24	G_0'
555	466	G_0'
565	73	G_0'
655	−442	
665	−179	
675	−47	
066 ·454	−43	
067 ·455	−116	
465	−58	

Argument-number.	Coefficient.	
07 (or Mf) group.		
071 ·755	26	
072 ·556	91	
073 ·545	98	
555	1370	
565	−88	
655	15	G_0'
074 ·554	−17	
556	48	
566	12	
075 ·345	−36	
355	677	
365	−44	
455	76	G_0'
465	12	G_0'
555	15642	
565	6481	
575	607	
585	−13	
076 ·554	−54	
564	−14	
077 ·355	−47	
365	−19	
08 group.		
081 ·655	42	
082 ·456	16	
656	26	
666	11	
083 ·445	22	
455	217	
465	−14	
555	13	G_0'
655	569	
665	236	
675	21	
084 ·456	28	
466	10	
555	−16	

Argument-number.	Coefficient.	
08 group—*contd.*		
085 ·255	54	
455	2995	
465	1241	
475	117	
555	38	G_0'
565	24	G_0'
675	−12	
086 ·454	−26	
09 group.		
091 ·555	20	
755	14	
092 ·556	32	
566	13	
093 ·355	25	
555	478	
565	200	
575	19	
095 ·355	396	
365	165	
375	16	
455	11	G_0'
0X group.		
0X1 ·655	23	
0X3 ·455	116	
465	48	
0X5 ·255	45	
265	19	
0E group.		
0E1 ·555	12	
0E3 ·355	19	

Diurnal



Development of the Tide-generating Potential. 323

SCHEDULE 1.

$G_1 = G\sin 2\lambda$, *associated with coefficients of sines to five decimals.*
$G_1' = 0{\cdot}72618\,G\cos\lambda\,(1-5\sin^2\lambda)$, *associated with coefficients of cosines to five decimals.*
(*When no geodetic coefficient is entered* G_1 *is understood.*)

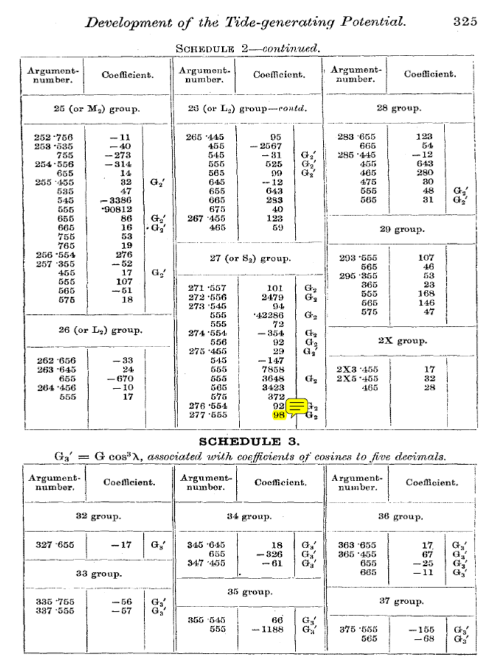
Argument-number.	Coefficient.	
10 group.		
105 ·955	11	
107 ·755	46	
109 ·555	28	
11 group.		
115 ·755	−10	G_1'
845	21	
855	108	
117 ·555	−10	G_1'
645	53	
655	278	
118 ·654	21	
119 ·445	10	
455	54	
12 group.		
124 ·756	−13	
125 ·645	−23	G_1'
655	−58	G_1'
745	180	
755	955	
126 ·556	−16	
655	−11	
754	15	
127 ·455	−11	G_1'
545	218	
555	1153	
128 ·544	14	
554	79	
129 ·355	35	
13 (or Q_1) group.		
133 ·855	−23	
134 ·656	−61	

Argument-number.	Coefficient.	
13 (or Q_1) group—*contd.*		
135 ·435	−28	
545	−84	G_1'
555	−211	G_1'
635	−42	
645	1360	
655	7216	
755	−13	G_1'
855	−19	
136 ·456	−13	
555	−39	
644	11	
654	68	
137 ·445	258	
455	1371	
555	−18	G_1'
655	−78	
665	24	
138 ·444	11	
454	64	
139 ·455	−14	
14 (or O_1) group.		
143 ·535	−17	
745	−20	
755	−113	
144 ·546	−15	
556	−130	
145 ·455	12	G_1'
535	−218	
545	7105	
555	37689	
645	16	G_1'
655	−108	G_1'
665	14	G_1'
755	−243	
765	−40	
146 ·544	12	
554	115	
147 ·355	−21	
455	−21	G_1'
545	14	
555	−491	
565	107	
148 ·554	−33	

Argument-number.	Coefficient.	
15 (or M_1) group.		
152 ·656	−14	
153 ·645	−63	
655	−278	
154 ·656	15	
155 ·435	17	
445	−197	
455	−1065	
545	98	G_1'
555	−661	G_1'
565	86	G_1'
645	85	
655	−2964	
665	−594	
675	17	
156 ·555	16	
654	−18	
157 ·445	16	
455	−566	
465	−124	
158 ·454	−24	
16 (or K_1) group.		
161 ·557	42	G_1
162 ·556	1029	G_1
163 ·535	14	
545	−199	
555	30	
555	17554	G_1
557	−11	G_1
755	−26	
164 ·554	−147	G_1
556	−423	G_1
165 ·455	−36	G_1
545	1050	
555	− ·16817	G_1
555	− ·36233	
565	−7182	
575	154	
655	−13	G_1'
166 ·554	−423	G_1
167 ·355	−26	
553	−11	G_1
555	−756	G_1
565	29	
575	14	
168 ·554	−44	G_1

Y 2

Semi-, and Ter-Diurnal



Development of the Tide-generating Potential. 325

SCHEDULE 2—*continued.*

Argument-number.	Coefficient.	
25 (or M_2) group.		
252 ·756	−11	
253 ·535	−40	
755	−273	
254 ·556	−314	
655	14	
255 ·455	32	G_2'
535	47	
545	−3386	
555	·90812	
655	86	G_2'
665	16	G_2'
755	53	
765	19	
256 ·554	276	
257 ·355	−52	
455	17	G_2'
555	107	
565	−51	
575	18	
26 (or L_2) group.		
262 ·656	−33	
263 ·645	24	
655	−670	
264 ·456	−10	
555	17	

Argument-number.	Coefficient.	
26 (or L_2) group—*contd.*		
265 ·445	95	
455	−2567	
545	−31	G_2'
555	525	G_2'
565	99	G_2'
645	−12	
655	643	
665	283	
675	40	
267 ·455	123	
465	59	
27 (or S_2) group.		
271 ·557	101	G_2
272 ·556	2479	G_2
273 ·545	94	
555	·42286	G_2
555	72	
274 ·554	−354	G_2
556	92	G_2
275 ·455	29	G_2'
545	−147	
555	7858	
555	3648	G_2
565	3423	
575	372	
276 ·554	92	G_2
277 ·555	98	G_2

Argument-number.	Coefficient.	
28 group.		
283 ·655	123	
665	54	
285 ·445	−12	
455	643	
465	280	
475	30	
555	48	G_2'
565	31	G_2'
29 group.		
293 ·555	107	
565	46	
295 ·355	53	
365	23	
555	168	
565	146	
575	47	
2X group.		
2X3 ·455	17	
2X5 ·455	32	
465	28	

SCHEDULE 3.

$G_3' = G\cos^3\lambda$, *associated with coefficients of cosines to five decimals.*

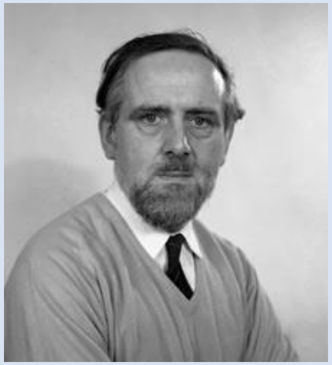
Argument-number.	Coefficient.	
32 group.		
327 ·655	−17	G_3'
33 group.		
335 ·755	−56	G_3'
337 ·555	−57	G_3'

Argument-number.	Coefficient.	
34 group.		
345 ·645	18	G_3'
655	−326	G_3'
347 ·455	−61	G_3'
35 group.		
355 ·545	66	G_3'
555	−1188	G_3'

Argument-number.	Coefficient.	
36 group.		
363 ·655	17	G_3'
365 ·455	67	G_3'
655	−25	G_3'
665	−11	G_3'
37 group.		
375 ·555	−155	G_3'
565	−68	G_3'

Time-domain Approach (Munk and Cartwright, 1966)

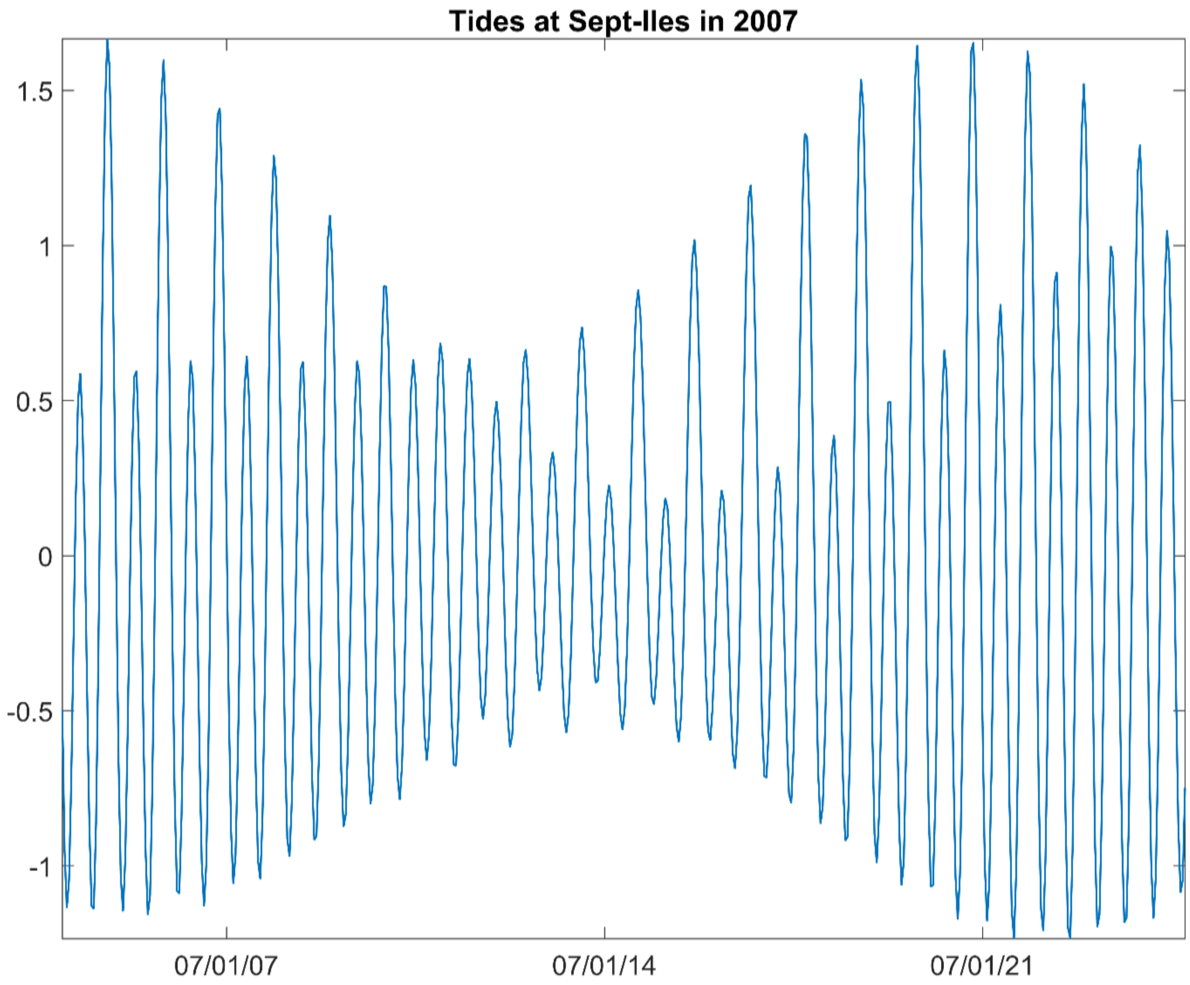
Munk

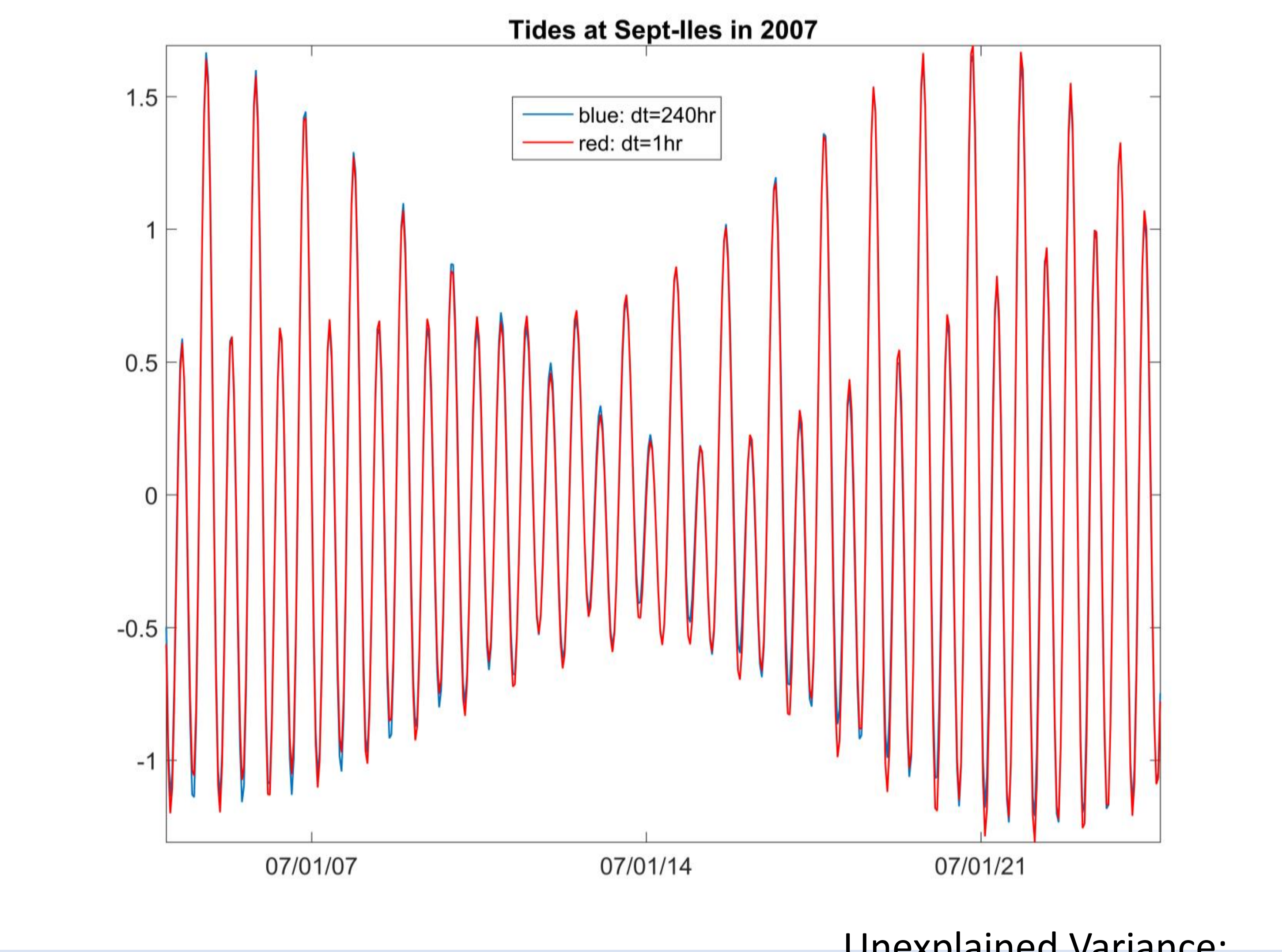


Cartwright

“We decided to take a new look at the tide records, without astronomical prejudice and freely allowing for the presence of noise. Modern methods of time series analysis seemed appropriate.

Nineteen years of hourly tide readings at Honolulu, Hawaii, and Newlyn, England, are analysed without astronomical prejudice as to what frequencies are present, and what are not, thus allowing for background noise.”


Tides at Sept-Iles in 2007
1.5
1
0.5
0
-0.5
-1
07/01/07
07/01/14
07/01/21



Unexplained Variance:
4.94% with dt = 10 days
4.59% with dt= 1 hour

Explained and Residual Variances

Explained:

$$\frac{(\eta-\overline{\eta})'(\eta-\overline{\eta})}{(\eta_{obs}-\overline{\eta_{obs}})'(\eta_{obs}-\overline{\eta_{obs}})}=0.9507=95.07\%$$

Residual:

$$\frac{\varepsilon'\varepsilon}{(\eta_{obs}-\overline{\eta_{obs}})'(\eta_{obs}-\overline{\eta_{obs}})}=0.0493=4.93\%$$

ASGF+SVD Method

Compared with the harmonic method (t_tide)

Harmonic Method

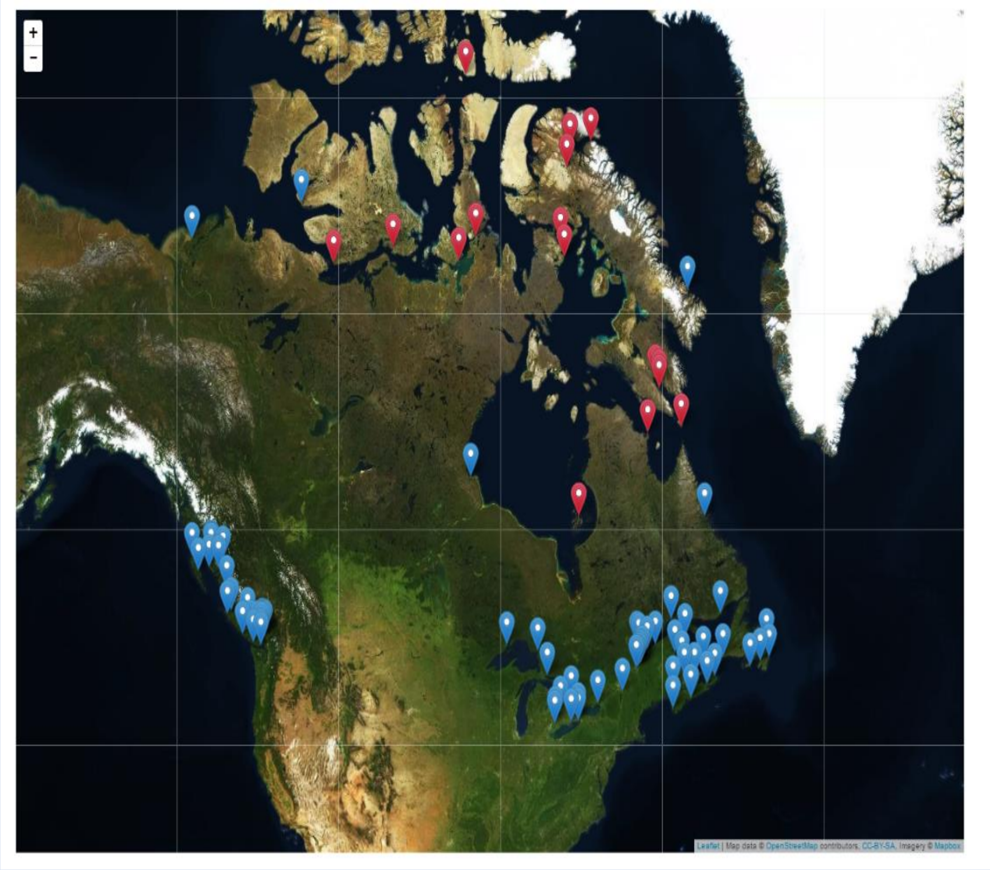
Decades	$\frac{\text{residual variance}}{\text{total variance}}\times 100\%$
1970-1980	5.19%
1980-1990	5.35 %
1990-2000	5.22 %
2000-2011	4.79 %
Mean	5.14%

VTG Work Plan

To establish a network of VTGs to cover three coasts in about 3 years:

1) To accompany the network of RTGs that are still operated by CHS.
2) To accompany the network of RTGs that have been abandoned by CHS.
3) To make good use of short term observations for VTG operations in Arctic.
4) To support OPP for additional POIs.
5) A VTG web page

Permanent and Temporary Tide Gauges



Locations of Tidal Measurement in Arctic Region (CHS Central & Arctic)

Permanent

Temporary, Bottom Pressure Sensor

Virtual Tide Gauge (VTG) Web Page:

http://142.130.48.193/test/vtg.php

http://odylab.pagekite.me/test/vtg.php

Summary

1. A VTG is a cutting-edge technology, integrating
    a) All-Source Green’s Functions and Data Assimilation
    b) NASA/JPL ephemerides astronomic forcing
    c) MERRA and GEM4 atmospheric forcing
    d) Real-Time Tide Gauge Data Streams
    e) Research and application come together tightly
2. A VTG is a good substitute or backup to an RTG, and costs much less!
3. A VTG provides good quality of hindcast and forecast. Uncertainty of predictions is reduced by 50% compared with traditional tidal harmonic forecasts.
4. A VTG webpage is being developed to distribute forecasts and hindcasts (requiring a public IP address!)

Thanks

- Thanks for Denis Lefaivre for sharing my research interest all the time.
- Thanks for Michel Beaulieu for his assistantship.
- Thanks for local CHS staff being interested in VTGs.
- Thanks all for your attending!